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Of Our Own Nation: John Wallis's Account of Mathematical Learning in Medieval England

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In *A treatise of algebra both historical and practical* (London 1685), John Wallis wrote the first survey of the state of mathematical learning in medieval England, and discussed with particular care the arrival and significance of the Hindu–Arabic numeral system. This paper offers a detailed commentary on Wallis's account in relation to the sources he used and the 17th-century Oxford context in which he wrote. The paper also supplements Wallis's treatment where possible with some of the findings of modern scholarship. It therefore provides on the one hand an overview of the spread of mathematical learning into medieval England, and on the other an insight into late 17th-century historiography. Wallis pioneered several new historiographical methods and can perhaps be claimed as the first modern historian of mathematics. © 2001 Academic Press

Dans *A treatise of algebra both historical and practical* (Londres 1685) John Wallis a été le premier à écrire sur l'etat des études de mathématiques dans l'Angleterre du Moyen Age, et il a discuté avec un soin particulier de l'arrivée et du sens du système numérique hindou–arabe. Cet exposé offre un commentaire détaillé sur l'essai de Wallis en ce qui concerne les sources utilisées et le contexte d'Oxford au dix-septième siècle, époque à laquelle il écrivit. Cet exposé ajoute en plus au traitement de Wallis certaines découvertes d'erudition moderne. Il fournit donc d'une part un aperçu général de l'étendue des études de mathématiques dans l'Angleterre moyenâgeuse et d'autre part un aperçu dans l'historiographie à la fin du dix-septième siècle. Wallis à été le pionier de plusiers méthodes d'historiographie et pourrait être acclamé comme le premier historien moderne de mathématiques. © 2001 Academic Press

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When John Wallis (1616–1703) wrote his great historical study, *A treatise of algebra both historical and practical* (1685), he devoted three-quarters of its 100 chapters to exploring the work of five 17th-century English mathematicians: Oughtred, Harriot, Pell, Newton, and himself [Stedall 2000]. His pride in the achievements of his own countrymen in his own century was from the beginning a prime reason for writing his book not just as mathematics but as history, a history designed to make the English contribution plain. The first 14 chapters of *A treatise of algebra* are a prelude to Wallis's larger plan: in them he explored the origins and early development of algebra and the evolution of the modern number system without which, he argued, algebra could never have progressed. But above all, Wallis began to set his story in an English context, by pointing to the existence of mathematical learning in England since early postclassical times, and especially during the later medieval period 1100–1450. Commentators on *A treatise of algebra* have tended to pass over these early

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chapters,[1] yet they reveal Wallis at his finest as a historian, for here he displayed greater objectivity and a truer sense of the complexities of historical development than in almost anything else he wrote, and in his investigations into the origins of the number system, he was the first to apply modern historiographical standards and methods to both primary and secondary mathematical sources.

A unique combination of circumstances in 17th-century Oxford made Wallis's research possible. Histories of mathematics are perhaps written only when mathematicians perceive marked changes in the nature and scope of their subject,[2] and by the second half of the 17th century it was plain that mathematics was steadily liberating itself from the constraints of its classical past and taking on a life and momentum of its own. Wallis had seen this revolution at first hand during his long tenure of the Savilian professorship, and indeed had done much to bring it about. He was also well placed in a second and more material way, through his access to the unprecedented accumulation of books and manuscripts in Oxford's Bodleian Library. From the opening of the library in 1602 there had been energetic and wide-ranging efforts to collect and preserve texts from England and abroad [Appendix I; Philip 1983] and the concentration of this wealth of material in a single place both reflected and encouraged new attitudes to historical study. The range of Wallis's reading will become evident in the course of this paper: his longstanding interests in grammar, etymology, cryptanalysis, music, astronomy, calendar reform, and general history all informed his account of the medieval period. He knew his classical sources thoroughly but also recognized, thanks to the new proliferation of oriental studies in Oxford [Russell 1994; Toomer 1996; Feingold 1997], the debt of European mathematics to Indian and Islamic sources, and the main theme of Chaps. 2–4 of *A treatise of algebra* was the transmission of learning from Islamic Spain to northern Europe. He had already published some of the material 30 years before in his *Mathesis universalis* [Wallis 1657, Chaps. 6–9] but there was also much in *A treatise of algebra* that was new.

This paper follows Wallis's Chaps. 2–4 in some detail and with a double purpose: first, to discover what was known and understood during the 17th century about mathematics in the medieval period; second, to look at Wallis's methods of research, and his establishment and use of new historiographical techniques. Each of Wallis's paragraphs, numbered for ease of reference, will be quoted in full followed by an accompanying commentary.[3] The story is taken up part way through Chapter 2, at paragraph 10, where having briefly considered what (slender) evidence of algebra could be prised from the writings of Euclid, Archimedes, Pappus, and Diophantus,[4] Wallis turned to Arabic mathematics.

[1] Wallis's chief biographer, J. F. Scott, disposed of the early chapters in one sentence: '[Wallis's] account of the history of mathematics in antiquity is very comprehensive and gives evidence of a close study of the classical literature of the sciences.' Scott made no mention of Wallis's researches on the medieval period [Scott 1936, 335; Scott 1938a, 133]. For commentary on selected paragraphs from Wallis's chapters 1, 2 and 6 see [Molland 1994, 215–218].

[2] The first history of mathematics was written by Eudemus (late fourth century B.C.) who, like Wallis in the 17th century, was aware of the many new discoveries made by his predecessors [Fauvel and Gray 1987, 46–47].

[3] Wallis made a number of handwritten corrections and annotations to his text in his own copy of *A treatise of algebra* (Savile A.3). Most are corrections of typographical errors and are incorporated in the transcripts given here without comment. Lengthier annotations are shown in curly brackets { }.

[4] The existence of a Greek "geometrical algebra" has been discussed at some length during the twentieth century: see, for instance [Unguru 1975, 1979; Van der Waerden 1976; Freudenthal 1977; Weil 1978; Mueller 1981, 43–44, 50–52; Berggren 1984, 394–410].

§ 2.10 After *Diophantus* (if not before, also) this learning was pursued by *Arabic* authors (but little known in *Europe* for a long time). From them it had the name of *Algebra*; not (as some would have it) from *Geber*, whom they conjecture (without any good ground that I know of) to have been its first inventer; but (as was said before) from its *Arabic* name, *Al-gjābr W'al-mokābala*.

The term "Arabic authors" here and throughout should be taken, as Wallis intended, to mean writers from anywhere in the Islamic world who used Arabic, the common language of Islamic culture.

Immediately apparent in this paragraph is Wallis's interest in word derivation. It was a longstanding preoccupation: his reputation as a mathematician has overshadowed the fact that in 1653 he published a highly regarded treatise on English grammar [Wallis 1653], a substantial chapter of which was concerned with etymology. He was correct in tracing the word *algebra* to the *Al-jabr wa'l muqābala*, the seminal text of Muhammad ibn Mūsā al-Khwārizmī (c. 770–850) [Karpinski 1915; Grant 1974, 106–111]. On the opening page of *A treatise of algebra* Wallis had already traced the meanings of the Arab words *gjābara* (to restore or set a broken bone) and *kābala* (to set one thing against another) [Wallis 1685, 2]. A literal translation of the Arabic title would be "Restoration and balancing," perhaps originally referring to the well known method of solving a quadratic equation by completing (*restoring*) a geometric square and comparing (*balancing*) the result with a known quantity, though later the terms were used of operations on equations [Saliba 1973]. The mistaken association of algebra with Geber (the 12th-century astronomer Jābir ibn Aflah) was made by Girolamo Cardano (1501–1576) who, in his list of 12 great scientists, noted Mahometius Mosis filius (al-Khwārizmī) as the inventor of algebra but supposed that as a result of his invention he took the name Geber [Cardano 1553, 1011f].[5] Wallis had earlier considered the possible identity of al-Khwārizmī and Geber, but seems to have confused Jābir ibn Aflah with the early ninth-century alchemist Jābir ibn Hayyān, and had been unable to draw any firm conclusion [Wallis 1685, 4];[6] in this paragraph he rejected such a hypothesis.

§ 2.11 Divers writers ('tis said) there are of *Algebra* in that Language, and from them (I suppose) the Denominations of *Diophantus* (if from him they learned it) came to be changed; and (beside the Denominations of Root, square, and cube,) that of *Sursolids* (first, second, third, &c.) introduced. But I rather think the *Arabs*, either of themselves, or from some others, had it long before *Diophantus*, and think this reckoning of *Powers* (by Sursolids, &c.) different from *Diophantus* {to be a good Argument for it}.

Wallis had already described Diophantus' method of naming powers as *Μονάς* (unit), *Αριθμός* (number), *Δύναμις* (power, or square), *Κύβος* (cube), denoted by μ, ς, δ, κ, $\delta\delta$, $\delta\kappa$, $\kappa\kappa$ and so on [Wallis 1685, 4; Heath 1931, 476–478]. This was an *additive* system

[5] The writers listed by Cardano were Archimedes, Aristotle, Euclid, Ioannes Scotus, Ioannes Suisset, Apollonius of Perga, Archytas Tarantinus, Mahometus Mosis filius, Alchindus, Heber [Geber] Hispanus, Galen, Vitruvius. Wallis read and annotated Cardano's list in the Savile Library copy, now Savile S.11. The identity of Geber is not obvious from Cardano's list but in his *Encomium geometriae* [Cardano 1663, IV, 443], Cardano referred to Geber as a writer on triangles and circles, a clear reference to Jābir ibn Aflah. See also [Cifoletti 1996, 128; Høyrup 1996, 113]. For other ascriptions of algebra to Geber see [Cifoletti 1996, 128–135].

[6] Wallis knew Jābir ibn Aflah's commentary on Ptolemy's *Almagest*, published at Nuremberg in 1534, in the copy now known as Savile X. 3, but despite this thought that Jābir ibn Aflah lived in the ninth century. The alchemical writings of Jābir ibn Hayyān (late eighth to early ninth century) were published in London in 1686. Wallis discovered additional information on al-Khwārizmi in [Abū 'l-Faraj 1663, 161; Eutychius 1656, II, 447], both translated by the Oxford Arabist Edward Pococke.

in which higher powers are expressed as sums of preceding ones. Wallis supposed that the Arab writers introduced the alternative *multiplicative* system [Wallis 1657, Chap. 11] in which powers were called "root" (R), "zensus" (from *census*, literally wealth or excess, for a square, denoted Z), "cubus" (C), "zenso-zensus" (ZZ), "first sursolid" (e.g., $\int Z$), "censo-cubus" (ZC), "second sursolid" (e.g., $b \int Z$), and so on,[7] where every prime power has to be given a new name and symbol. This was the system generally used by the 16th-century cossist writers, and only with the rediscovery of Diophantus did the additive system come back into use alongside it (leading to potential confusion as to whether A_{qc} meant A^5 or A^6). Wallis later in *A treatise of algebra* claimed that the multiplicative system was used by "all our European Algebrists before Vieta, having learned it from the Moors" [Wallis 1685, 91], but in this he was mistaken: both systems were used in 15th-century Italy in the earliest attempts to deal with powers higher than 3 [Reich 1994].

> §2.12 With the *Arabs* all sorts of Mathematical Learning flourished, and was improved, for a long time together, while in *Europe* it was very much neglected. Amongst whom were *Maimon, Almeon, Alchindus, Albumasar, Alfraganus, Alfarabius, Geber, Mahometes Bagdadinus, Mahometes ben Musās, Thebit, Haly, Alchabitius, Alhazen*, and divers others. To whom I may add also some *Persians* and *Tartars*, as *Al-suphi, Nasir-eddin, Shah-colgius, Uleg-beig*, &c. whose Astronomical Tables are yet in being.

At several points in his account Wallis presented, as here, a list of names with little additional information: in this case there is not even a date. In every case such lists were drawn from Wallis's main source for this period, the *De scientiis mathematicis* of John Gerard Vossius (1577–1649), the third book in his trilogy on contemporary arts and sciences [Vossius 1650]. Vossius was a renowned Dutch scholar who had spent some years in England from 1629 to 1633 (and had been a canon of Canterbury) but had returned to take up the chair of history in the new university of Amsterdam. There he knew the English mathematician John Pell, who taught in Amsterdam from 1643 to 1646, and it may have been Pell who introduced the work of Vossius to Wallis. In *De Scientiis* descriptions and histories of different branches of mathematics (arithmetic, geometry, logistics, music, optics, and so on) are followed by an extensive chronological list of mathematicians, for each of whom Vossius gave, as far as possible, a date and details of extant works. Vossius was not, however, a mathematician and did not discuss mathematical content. As he said himself [Vossius 1650, 37]: *Neque enim ipsam tradimus scientiam; sed de ea scribimus* ("Nor do I teach the science itself, but only write about it"). His information was drawn from other authorities (whom he cited frequently); his own contribution was to collate and order the facts at his disposal.

Wallis's copy of *De scientiis* is preserved in the Bodleian Library and his frequent and detailed annotations show how thoroughly he read it. The Arabic writers listed in §2.12 are all to be found in *De scientiis* though not in the order given by Wallis. The first to appear is *Mahometes ben Musas* (al-Khwārizmī, c. 770-850), whom Vossius mentioned briefly at the end of his account of Greek and Latin writers on arithmetic [Vossius 1650, 41]. Vossius seems to have used Cardano as his source here: both referred to al-Khwārizmī as Mahomet son of Moses (*Mahomet Mosis filius*), and Vossius stated that Cardano listed Mahomet ninth (actually eighth) in his list of 12 great scientists.[8] A few pages later *Mahomet Bagdadinus*

[7] The exact symbolism varied from writer to writer but this scheme, from Recorde 1557, is typical.

[8] See note 5. Together with Mahometes ben Musas, Vossius also mentioned one Abraham Cai, a Jew. In the index to *De Scientiis* Abraham instead of Mahomet is wrongly described as the inventor of algebra and ninth in Cardano's list. Wallis made a handwritten correction in his copy.

(al-Baghdādī, fl. c.1230) and *Alchindus* (al-Kindī, c. 801–c. 866) were described by Vossius as writers on geometry [Vossius 1650, 61],[9] and then *Alchabitius 1410* (al-Quabīsī, fl. c. 950) and *Alhazen 1100* (al-Hasan or ibn al–Haytham, 965–c.1040) as writers on optics [Vossius 1650, 109].

The remaining writers, *Maimon 827* and *Almeon 838* (both references to Caliph al-Ma'mūn, 809–883, founder of the House of Wisdom in Baghdad),[10] *Albumasar 884* (Abū-Ma'shar, c. 810–886), *Alfraganus 879* (al-Farghānī, d. 861), *Alfarabius 940* (al-Fārābī, c. 870–950), *Geber* (Jābir ibn Aflah, fl. 1145), *Thebit 1300* (Thābit ibn Qurra, 836–901), and *Haly 1202* (Abū-'l-Hasan, fl. 1020–1040) are all listed as astronomers [Vossius 1650, 173–181]. Also in Vossius but curiously missing from Wallis's list are *Albategnius 888* (al-Battānī, 850–929), *Arzachel 1080* (al-Zarqualī or Azarquiel, d. 1100), and *Abenezra 1145* (Rabbi Abraham ben Meir ibn Ezra, 1092–1167). As Wallis mentioned all three later in connection with their astronomical tables, their exclusion from this preliminary list was perhaps deliberate. There are no other omissions: every Arab writer recorded by Vossius was also noted by Wallis. In Wallis's copy of *De scientiis* the pages on Arab astronomy are particularly heavily annotated[11] and it is clear that he read them carefully. But he also extracted Arab writers from other sections and ordered the entire list more or less chronologically for his own text.

Wallis's brief list of "Persians and Tartars" came from a different source. In 1648 John Greaves, a scholar of both Persian and Arabic, and Savilian Professor of Astronomy (1643–1649), had published the geographical tables of the Persian Nasīr al-Dīn al-Tūsī (1201–1274) and of the "Tartar" Ulugh Beg (1394–1449), king of Samarkand and founder of its observatory [Greaves 1648].[12] Two years later Greaves translated and published astronomical and chronological tables of Ulugh Beg, and the astronomy of the Persian al-Kāshī (Shah-colgius) (d. 1429) who assisted Ulugh Beg in Samarkand and made improvements to the astronomical tables of Nāsir al-Dīn al-Tūsī [Greaves 1650a; 1650b]. The catalogue of fixed stars compiled by Ulugh Beg also drew on the earlier observations of Al-Sūfī (903–986).

To the modern reader Wallis's list of names, devoid of historical context, raises far more questions than it answers, but Wallis was following a long established paradigm of historical writing, which concentrated on authors rather than ideas, and on stability rather than change. The underlying assumption, though one that Wallis could no longer completely share, was that mathematical knowledge derived from divine revelation or ancient authority, so that the history of mathematics was essentially the handing on (*traditio*) of such knowledge from one generation or culture to the next.

There are several histories of this kind by medieval and Renaissance writers. Assertions that mathematics was handed from the Babylonians or the Hebrews to the Egyptians and thence to the Greeks can be seen in the earliest post-Classical histories, those of Isidor of

[9] The *De superficierum divisionibus* of al-Baghdādī was published by Commandino in 1570 from a manuscript supplied by John Dee, who conjectured that it was a lost book of Euclid, though there is in fact only an indirect connection with Euclid's work.

[10] Vossius mentioned a second, later, *Maimon* who can be identified as the philosopher Rabbi Moshe ben Maimonides (1135–1204). Wallis omitted him, perhaps incorrectly assuming duplication.

[11] Page 177, in particular, from al-Farghānī to ibn Ezra, is heavily annotated at every paragraph.

[12] The Tartars came originally from the east Asian steppes; the description is probably used indiscriminately here for the various Mongol tribes that overran central Asia in the early thirteenth century.

Seville (570–636) [Isidor, III; Migne LXXXII, cols. 153–184 (155–169)] and Bede (672–735) [Migne XC, cols. 647–653 (650)]. Later medieval accounts became more sophisticated but presented much the same story. By the 13th century Bacon saw the history of science as a process of decline in which ancient knowledge was occasionally recovered only to be lost again [Molland 1995, 214, 221–223]. Two centuries later, in 1464, Regiomontanus wrote a history of mathematics in which the main theme was not change, but the continuity and stability of mathematics as handed from one mathematician to another [Regiomontanus 1464; Rose 1975, 95–98]. Cardano's 1553 list of 12 great writers has already been mentioned: there six Greeks, two Britons, a Roman, and three Arabs are ordered by eminence rather than chronology, and without any suggestion of historical development or context. Bernardino Baldi (1553–1617), in his *Vite de matematici* written almost at the end of the 16th century [Baldi 1998; Rose 1975, 253–269; Moyer 1999], still conveyed mathematics as a continuous tradition running from the Babylonians (Chaldeans) and Egyptians through the Greeks, Romans, and Arabs to his own time, in which the greatest achievement was the restoration of Archimedes, and his *Cronica de matematici* listed an unbroken line of mathematicians from 600 B.C. to 1596 [Baldi 1707]. A similar list, which was available to Wallis but which he appears not to have used, was compiled by Henry Savile in 1570 and is preserved in the Savile Library. Savile's list began with the sons of Seth and continued through the Druids and Zoroastrians to Abraham, Joseph, Homer, and Pythagoras before reaching the firmer historical ground of Classical Greece [MS Savile 29; Goulding 1999, 123–125].[13] Savile, like his predecessors, was chiefly concerned with demonstrating the deep roots of mathematics in its Classical past.

Only very gradually did there begin to emerge ideas of mathematical progress, a sense that modern mathematicians could add to or even improve upon the existing body of knowledge [Zilsel 1945; Lilley 1958, 3–37; Crombie 1975; Molland 1978; Molland 1983, 141–148]. Writing in the 1640s, Vossius still presented the various branches of mathematics as existing largely independent of age or culture, so that detailed tracing of ideas was less important than identifying the carriers of the tradition, who did not themselves need to be innovators. Wallis's lists of names, here and elsewhere, were quite compatible with this established style of historical writing; it makes his new methodology later all the more remarkable.

> § 2.13 From those *Arabians* we have the names of *Almagest, Azimuth, Almicanter, Zenith, Nadir, Almanack, Algorism, Algebra,* &c. and divers other *Arabic* words (now disused) we find retained in *Regiomontanus, Purbachius* and others before them, who either translated *Arabic* Authors, or at least derived their Learning from them. As I find in divers of those Manuscript Authors, which I have seen, concerning the *Astrolabe* (whose Parts they describe by *Arabic* names), and other Mathematical Learning.

Here again Wallis's interest in etymology is evident and he was correct in tracing all these words to Islamic scientific, and especially astronomical, writing. Georg Peuerbach (1423–1461), humanist and astronomer of the University of Vienna, was the teacher of Johannes Muller, Regiomontanus (1436–1476). They were regarded by their contemporaries as being responsible for the renaissance of astronomy in Europe, and Regiomontanus completed a translation and critique of Ptolemy's *Almagest* begun by Peuerbach [Regiomontanus 1550; Rose 1975, 90–94]. The Savile Library held a copy of the 1550 edition which would have been known to Wallis.

[13] Savile did present sound historical arguments about the identity of Euclid; see [Goulding 1999, 96–103].

§ 2.14 They translated *Euclid, Ptolemy, Aristotle*, and divers others of the *Greek* Authors into *Arabic*; and out of the *Arabic* we had our first Translations of *Euclid, Ptolemy*, and other *Greek* Authors, into *Latin*, before those out of the *Greek*. A thing of it self notorious, and so also attested by *Vossius*, (after Sir Henry *Savil*:) *Euclidem Latini Translatum habuerunt prius ex Arabico quam ex Graeco fonte. Quemadmodum & ante CC, & infra, annos, non alia Aristotelis, Galeni, Ptolemaei, aliorumque multorum, interpretatio in manibus erat, quam ex Arabica versione Latine, vel Semibarbaro, potius, expressa*. And by Sir *Henry Savil*, in his second Lecture on *Euclid*, almost in the same words. And from them we received not only our *Algebra*, but other parts of Mathematical Learning; brought by the *Moors* into *Spain*, and from thence propagated to other parts of *Europe*; about the year of our Lord 1100, or somewhat sooner.

The passage from Vossius translates as: "They had a Latin translation of Euclid from Arabic before any from a Greek source. Just as, for up to two hundred years before that, they had no translation of Aristotle, Galen, Ptolemy or many others, other than Latin, or rather semibarbarous, versions from Arabic." Vossius did not acknowledge Savile as the source of his information but Wallis clearly knew Savile's 1619 lectures on Euclid well, and recognised the relevant passage [Savile 1621, 35]:[14] *Et quidem nos occidentales Europaei Arabibus primus omnium debemus Aristotelem, Euclidem, Galenum, Ptolemaeum, caeteros Graecorum Principes, cum ante centum annos aliae versiones nullae, praeterquam ex Arabico, fuerint in manibus nostrorum hominum, quod Graecae linguae cognito nondum in Italiam et Occidentem immigrasset*. Wallis assumed that this information was already well known (notorious).

Wallis was less than careful here and elsewhere in distinguishing between different periods and geographical locations of Islamic culture, and probably used "Moors" in the general sense of "Muslims." Mathematical learning was brought not only by the true Moors, the north African invaders and settlers of Spain, but also by later travellers and scholars from elsewhere in the Islamic world, from the old Hellenistic regions of the eastern Mediterranean and the new centre of learning at Baghdad [Fletcher 1992].

The first Latin translations of Euclid from Arabic were made by Adelard of Bath (c. 1130), Hermann of Carinthia (c. 1143), and Gerard of Cremona, who also did the first translation from Arabic of Ptolemy's *Almagest* (c. 1175). In fact a translation of the *Almagest* directly from Greek was done in Sicily in 1165 by an anonymous student from Salerno who has also been credited with a translation of Euclid's *Elements* from Greek, but neither translation was well known until the 20th century and they would not have been known to Savile, Vossius, or Wallis [Rose 1975, 76–79].

§ 2.15 Upon this account, I find that divers of our own Nation, about the twelfth and thirteenth Century, (not satisfied with the Philosophy of the Schoolmen,) were inquisitive into the *Arabic* Language, and the Mathematical Learning therein contained.

Wallis was here, perhaps deliberately, pointing to the medieval forebears of the 17th-century Oxford interest in Arabic language and science. He was also introducing two new themes he was about to explore in detail. A more precise determination of the date of transmission of Arabic learning through Spain to the rest of Europe was to be the subject of his next two chapters. The role of Englishmen in acquiring and propagating the new ideas, and the subsequent revitalisation of English mathematics fills the remainder of Wallis's Chapter 2.

§ 2.16 As *Adelardus*, (a monk of *Bath*) whom *Vossius* placeth about the year 1130 who for that purpose travelled into *Spain*, *Egypt*, and *Arabia*; and (as *Vossius* tells us) translated *Euclid* (and some other *Arabic*

[14] Savile presented the Bodleian Library with an inscribed copy of his *Praelectiones*, now 4^0 S.39 Art; Savile's original manuscript is also preserved, as MS Savile 37.

> authors) out of *Arabic* into *Latin, Anno hoc MCXXX. Athelardus sive Adelardus, Anglus, Monachus Bathoniensis, Euclidis Geometriam ex Arabico vertit Latine. Nec, Arabice scivisse, mirandum: Quando non modo Galliam, Germaniam, Italiam, adiit; sed etiam Hispaniam, Ægyptum, Arabiam ipsam.*

The quotation from Vossius translates as: "The year 1130 Athelard, or Adelard, an Englishman, a monk of Bath, translated the geometry of Euclid from Arabic to Latin. Nor is it any wonder he knew Arabic when he had been not only to France, Germany, and Italy but also to Spain, Egypt, and Arabia itself." Modern scholarship has modified this account: Adelard (c. 1080–1150) travelled widely in France, Sicily (before 1116), Cilicia (in what is now eastern Turkey), Syria, and Palestine but there is no firm evidence that he ever visited Spain [Burnett 1987]. He is best known for the first translations of Euclid from Arabic to Latin, and three versions are ascribed to him.[15] Wallis knew one version in the Savile Library and another in Trinity College.[16] Adelard also translated the astronomical tables of al-Khwārizmī, *Ezich elkaurizmi*; the Bodleian Library owns a copy that is richly and beautifully illustrated in red, green, and gold [MS Auct F.19].

> § 2.17 And *Robertus Retinensis* (*Robert of Reading*) who travelling into *Spain* on the account of the *Mathematics*, did there translate the Alcoran out of *Arabic* into *Latin*, in the year 1143. (As appears by his Epilogue to that Translation, and the Preface of *Petrus Cluniacensis* thereunto.)

There is no mention of Robert in the pages of *De scientiis* and his inclusion here is a result of Wallis's own researches among Bodleian manuscripts: a copy of Robert's translation of the *Koran*, made for Peter, Abbot of Cluny, was acquired by the library as part of the Selden collection [MS Selden Supra 31, ff. 32–204].[17] In the preface (which appears in the Bodleian manuscript as a colophon or endpiece) Robert wrote that he now intended to return to his chief interest, mathematics [MS Selden Supra 31, ff. 32–33; Migne CLXXXIX, col. 657f], and for Wallis this was reason enough to count him among the English translators of Arab mathematical learning.

Robert's name actually appears at the end of the translation as *Ketenensis* but the looped 'K' was read by Wallis as 'R.' Robert was in fact *Robert of Chester* whose name has mutated through the forms Cestrensis, Kestrensis, Ketenensis, and Retinensis leading to confusion which persists to the present day: the Bodleian Library catalogue entry for the manuscript describes the author as "probably of Ketton in Rutland" (whereas Wallis translated "Retinensis" as "of Reading"). The *Dictionary of national biography* still carries two articles on Robert, headed "Chester, Robert (fl. 1182)" and "Robert the Englishman, (de Ketenes, de Retines) (fl. 1143)." The date 1182 in the former arises from the dating system then in use in Spain; it was in fact the year we would now denote as 1144, which at least brings the two Roberts into the same time frame.

Little is known of Robert's life. He was in Spain from about 1140 and lived near the river Ebro in the north east. He worked closely with another translator, Hermann of Carinthia, who appears to have come from the region that borders modern Austria and Slovenia (he is also sometimes known as Herman of Dalmatia), which suggests a trans-European dimension

[15] Adelard I is a close translation of the entire work; Adelard II was the most popular version but omits many of the proofs; Adelard III is a commentary rather than a translation. Recent scholarship has questioned the true authorship of Adelard II and has suggested that it should be ascribed to Robert of Chester; see [Busard and Folkerts 1992].

[16] MS Savile 19 (Adelard II); MS Trinity College 47 (Adelard I).

[17] Peter commissioned the translation so that he could refute Islam; see [Migne CLXXXIX, col. 649f; col. 659f].

to the translation programme that Wallis either failed to see or chose to ignore [Thorndike 1923, II, 14–98, 155–187; Haskins 1924, 3–66, 113, 129; Rose 1975, 76–89; Lindberg 1978, 52–90]. As a translator Robert was far more important than Wallis knew. By misreading his surname Wallis failed to recognise him as the translator of the *Canons* of Arzachel (to be discussed in § 4.6). Robert is now also thought to be the possible writer of "Adelard II" [Busard and Folkerts 1992]. He is best known, however, for the first Latin translation of al-Khwārizmī's *Al-jabr wa'l muqābala*, the key text in the evolution of Arabic and European algebra. No copy of Robert's translation reached England [Karpinski 1915, 49–63; Hughes 1982], and unfortunately Wallis never knew of this important English contribution to the early development of algebra.

> § 2.18 About the same time (or somewhat sooner) *Guilielmus de Conchis (William Shelley)* is said to have travelled into *Spain* to furnish himself with *Arabic* and *Mathematical* Learning; and brought from thence divers *Arabic* Books.

Guillaume de Conches (d. ?1154) was a natural philosopher, born in Normandy.[18] He studied at Chartres, taught at Chartres or Paris, and retired to Anjou where he wrote the philosophical work for which he is best known, his *Dragmaticon*. There is no evidence that he ever went to Spain, nor that he was familiar with Arabic language or philosophy, or with astronomical tables of any kind. Nor was he mentioned by Vossius. His inclusion by Wallis is therefore puzzling until we look at the next name, Daniel Morley.

> § 2.19 And, soon after, *Daniel Merlacus (Morley)*, about the year 1180 made several Journeys into *Spain* on the like account, where (at *Toledo*) *Arabic* and *Mathematical Learning* were in great request (brought thither by the *Moors*) which in other parts of *Europe* were scarce known. And these brought with them that kind of Learning into *England* very early, with store of *Arabic* Books.

This information about *Daniel Morley* (fl. 1170–1190) is not to be found in *De scientiis*, but in the preface [Halliwell 1839, 84–85] to Morley's only work, his *Liber de naturis inferiorum et superiorum*. There Morley helpfully gave a brief account of his life and travels which tallies with the summary given by Wallis. First, he said, he went to Paris but found that the teachers there carried only "leaden pens" with which they marked asterisks and obelisks reverently in their texts, so he went on south to Toledo in search of something better, in particular the contents of the *quadrivium*, the four classical branches of mathematics. On returning to England with a good collection of books, he was depressed by the neglect of Plato and Aristotle there, and decided to return to Spain, but was waylaid by John, Bishop of Norwich (1175–1200), for whom he wrote his treatise. The only manuscript copy of the preface now surviving is in the British Library [Sudhoff 1918; Birkenmajer 1970, 45–51], but Wallis knew of another copy that had been in Oxford a few years earlier:

> § 2.20 A particular account of these Travels of *Shelley* and *Morley* was a while since to be seen in two Prefaces, to two Manuscript Books of theirs in the Library of *Corpus-Christi* College in *Oxford*, but hath lately (by some unknown hand) been cut out, and carried away; which Prefaces (one or both of them) did also make mention of the Travels of *Athelardus Bathoniensis*, and are, to that purpose, cited by *Vossius* out of the Manuscript Copy. Who ever hath them, would do a kindness (by some way or other) to restore them, or at least a Copy of them.

[18] It was the English historian John Bale (1506–1552) who claimed that Conches was born in Cornwall and who introduced the anglicized form of his name, Shelley.

The Corpus Christi manuscript to which Wallis referred is that now known as MS CCC 95. It includes a copy of the *Liber de naturis* from which the preface has, as Wallis described, been neatly cut out, but the contents page lists the opening work as *Philosophia magistri Daniel de Merlac*. Morley's book is followed without a break by the *Dragmaticon* of de Conches, which ends "Explicit Will de Conchys," an attribution which led to a mistaken identification in Henry Coxe's 1852 catalog [Coxe 1852].[19] Wallis (or his unknown informant) must have been similarly misled, and Wallis assumed that the missing preface described a journey taken by both men, even though he dated them 40 years apart.

The preface relating to Adelard comes from a different Corpus Christi manuscript, MS CCC 86, containing Adelard's *De causis* [MS CCC 86, f. 163]. It was used by Vossius to date Adelard's activities to 1130, and Wallis would have known of it from the reference in *De scientiis* [Vossius 1650, 176].

> § 2.21 About the same time were *Johannes Sarisburiensis*, *Rogerus Infans*, and divers others of the *English*.

John of Salisbury (d. 1180) was one of the best known scholars of the day. He travelled as far as southern Italy, but knew little Greek and no Arabic, and employed an Italian Greek to make translations of Aristotle. He was primarily a theologian and no lover of mathematics, which to him meant astrology: in his *Polycraticus* he defined mathematicians as those "who from the position of the stars and the motion of the planets foretell the future," and classed mathematics with chiromancy, sortilege, and augury as one of the magic arts, and a source of evil [Migne CIC, cols. 407–409].

Roger Infans was the scholar *Roger of Hereford* (fl. 1178), but Wallis never used the second, more usual, form of his name. Roger was a natural philosopher, computist, and astrologer, with special knowledge of mines and minerals, and was familiar with some Arabic texts, but it is not known whether he made his own translations [Russell 1932; French 1996]. There is no mention of him in *De scientiis* and Wallis must have come across the unique occurrence of "Infans" in MS Digby 40, one of the few instances where we can be sure that Wallis consulted the Digby collection. Roger's *Tractatus de computo* in MS Digby 40 is headed "Tractatus Rogeri Infantis," apparently because Roger said that he wrote it while still a young man. As a result, he, like Robert of Chester, has acquired two entries in the *Dictionary of national biography*: "Roger Infans (fl. 1124)" and "Roger of Hereford (fl. 1178)." "The mismatching dates stem from the figure 1124 which appears in the margin of the *Tractatus de computo*, but which was meant as part of the calculation, not as a date of writing. Roger's name was anglicized by the historian John Leland (1506–1522) to "Yonge;" Wallis in the 1693 Latin translation of *A treatise of algebra* went further and gave his name as "Roger Child" [Wallis 1693, 6].

[19] The manuscript is described by Coxe 1852 as "three books of the Norman philosopher William de Conches, alias Shelley." The error was pointed out by H. Nash in a letter to Corpus Christi librarian, Charles Plummer, written 25 March 1889. The letter (preserved with MS CCC 95) begins: "I have been to see the BM MS (Arundel 377) of Daniel de Morley. It is the same book as the one in your library and it is then also followed by a dialogue between the Duke of Normandy (D) and the Philosopher (P) of Gul. De Conchis. Coxe confounded the two. A passage which I copied from your MS fol 15b occurs on the last folio of the Arundel 377, where the division between the two is quite distinct. You will see the 'incipit' of Gul de Conchis in the Arundel catalogue. I mention this as you may like to make a note of the fact in your copy of Coxe's catalogue. The beginning (missing in the CCC MS) contains a delightful little piece of autobiography."

> § 2.22 Before these times the *Arabic* Language, and *Greek* it self, being but little known in these Parts, Mathematical Learning was but very rare, and slenderly improved in *Europe*. We had indeed in *England*, *Althelmus* or *Adelmus*, whom Vossius placeth about the year 680; and *Walfridus Ripponensis*, placed by him at 690; and *Bede* (the most eminent of that Age) at 730; and *Albinus* or *Alcuinus*, (a Scholar of *Bede*) at 760; but *Euclid* and *Ptolemy* were unknown to them, *Boethius* and St. *Augustin* being their most Classic Authors for such Learning.

Wallis recognized that during the period when Greek was lost in Europe and Arabic not yet understood, there was little mathematical learning of any significance. *Boethius* (480–524 A.D.), who witnessed the death throes of the Roman empire in the west, based his *Arithmetica* on the earlier *Introductio arithmeticae* of Nicomachus (c. 100 A.D.), essentially a treatise on Pythagorean number relationships. As a mathematician Boethius was no more than a pale shadow of the great Classical writers, but in an age when, as Wallis described, Euclid and Ptolemy were almost completely lost, he was one of the few remaining links to the Greek mathematical past and his *Arithmetica* was copied and used for centuries [Evans G.R. 1978; Masi 1983; Oosthuit and Schilling 1999].

It is more difficult to justify the inclusion of *St. Augustine* as an upholder of Classical mathematics. Wallis, however, annotated his copy of *De scientiis* with a reminder of the use of mathematics in theology and quoted Augustine: "*nemo ad rerum divinarum, humanorumque, cognitionem accedat, nisi prius numerandi artem addiscat*" ("no one can attain knowledge of things divine or human unless he first learns also the art of numbering") [Savile G. 21, 30–31]. In *The city of God* Augustine argued that the science of number was an aid to interpretation of the scriptures, and speculated that the universe was created in six days because six is a perfect number [*De civitate dei*, 11, Ch. 30; Migne XLI, cols. 345–346]. It seems, though, that Wallis had something more practical in mind for he referred to the use of mathematics in the calculation of chronology. More generally, the correct measurement and division of time, an art known as *computus*, was extremely important in a society increasingly concerned with the correct regulation of religious life and festivals, and served to keep some advanced arithmetic alive during the early medieval era. Three of the four English scholars mentioned by Wallis (Aldhelm, Wilfrid, and Bede) were renowned computists.

Aldhelm (640–709), Abbot of Malmesbury and Bishop of Sherborne, was educated at Malmesbury and Canterbury in law, computation, and astronomy, and wrote sophisticated Latin [Vossius 1650, 171, 312, 395; Migne LXXXIX]. He was the author of *Liber de septenario*, a treatise on the number 7, but it was a mystical rather than mathematical work. His reputation for mathematics arose not from this but from the quarrel between the Celtic and Roman churches over the calculation of the date of Easter in which Aldhelm was a proponent of the Roman method, based on the 19-year lunar cycle. His exact contemporary, *Wilfrid of Ripon* (634–709), Archbishop of York, was instrumental in getting the Roman method accepted at the synod of Whitby in 664 [Vossius 1650, 395; Migne XCV].

Bede (672–735) was by far the most prolific scholar of the period [Vossius 1650, 171, 312; Migne XC; Jones C. 1970]. He spent all his life at the monastery of Jarrow-on-Tyne, Northumberland, which for a brief time was a focus of learning collected from Ireland, continental Europe, and even north Africa. Most of Bede's writing was on theology and history, but he also wrote a *Computus* [Migne XC, cols. 277f, 293f]. Bede's work became known on the continent through his pupil *Alcuin* (or Albinus) (735–804) who became an adviser to Charlemagne, and was the fourth of the scholars mentioned by Vossius and Wallis

[Vossius 171; Migne C, CI]. Alcuin encouraged the study of mathematics and the computus and is often credited with a set of 53 arithmetic and geometric puzzles, the "Propositions for sharpening the minds of youth" [Migne CI, cols. 1143–1160; Folkerts 1978; Singmaster and Hadley 1992].

Thanks to Alcuin, Bede's influence survived longer in continental Europe than it did in Britain. (The best manuscript of Bede's *Computus* in the Bodleian Library comes not from England but from France [MS Bodl. 309, ff. 3^{v}–62, 68–80].) In England, Bede's learning was never more than a fragile candle in a vast surrounding darkness, and it was all but extinguished in the invasions and instability of the three following centuries. Only early in the 12th century did scholars in England and elsewhere across Europe become aware of the knowledge that all this time had been accumulating in Islamic Spain, and some of the more adventurous travelled south and brought back texts that were to set the intellectual life of northern Europe on a new course.

> § 2.23 But after these times, having received from the *Arabs* divers Translations of *Euclid, Ptolemy, Aristotle* and other *Greek* Authors, with divers improvements in Philosophy, Astronomy, Geometry and other parts of Mathematics, these Studies were strangely advanced, and especially in England, where (beside those above mentioned) we had *Clement Langthon*, whom *Vossius* placeth about 1170; *Gervasius Tilburiensis*, about 1210; *Johannes de Sacro Bosco*, about 1232; *Robertus Lincolniensis* (*Robert Grosthead*) about the same time; *Roger Bacon*, about 1255; *Johannes Peccam* (or *Johannes Cantauriensis*) about 1276; *Odingtonus*, about 1280; *Johannes Bacondorpius*, about 1330; *Robert Holcot* (or *de Northamptona*) about 1340; *Johannes Estwood* (*de Ashenden*), about 1347; *Climitonus Langley*, about 1350; *Nicolaus Linnensis*, about 1355; *John Killingworth*, about 1360; *Richard Lavingham*, about 1370; *Simon Bredon*, about 1386; *John Sommer*, about 1390; *John Walter*, about 1400; *William Batecombe*, about 1410; *William Buttoner*, about 1460; who were, many of them, very eminent, as in other kinds of Learning, so particularly in the Mathematics; and divers of their Works are extant in our Libraries, which have not yet been printed.
>
> § 2.24 Besides others whom *Vossius* mentions not: As *Adamus de Marisco* (*Adam Marsh*), contemporary with *Grosthead* Bishop of *Lincoln*, intimate with him, and commended by him; *Bradwardine* and *Read*, and divers others about that Age.

Wallis began with a generous recognition that the Arabs had not only preserved and translated the Classical heritage but had developed and improved upon it, a sign of Oxford's new respect for Arabic learning, in contrast to European attitudes in earlier centuries [Rose 1975, 262–263; Moyer 1999, 480–481]. The influx of new texts from Arabic had revolutionized learning throughout western Europe, but Wallis was concerned only with England, and justified his claim that mathematical studies moved forward "especially in England" by producing a long list of 13th- and 14th-century English "mathematicians." The list was compiled by the same method Wallis had used for his Arab writers, by combing the pages of *De scientiis* for the names of every English writer he could find and then arranging them in chronological order (according to the dates given by Vossius). Vossius in turn had gathered his information on these writers from the earlier researches of the English historians John Leland, John Bale, and John Pits, all of them assiduous collectors of information on medieval writers and manuscripts [Appendix II].

Wallis was indirectly, therefore, using the best available evidence of the time, much of it collected during the 16th century from the libraries of Oxford, Cambridge, London, and Norwich, and from the monasteries at the time of their dissolution. To the modern reader, however, the list is a curious mixture of names well known and obscure, with widely varying

claims to mathematical prowess.[20] Langthorn, Tilbury, and Lavenham would hardly have thought of themselves as skilled mathematicians and are certainly not so remembered now. On the other hand there are some surprising omissions. Henry Savile in 1570 had classed the medieval mathematicians Richard Swineshead, Roger Bacon, and Richard Wallingford on a par with Archimedes and Ptolemy [MS Savile 29, f. 3^{v}] but by the 17th century Swineshead and Wallingford had slipped into oblivion.

Wallis's omission of Swineshead is particularly unaccountable since he knew something about him: Vossius in *De scientiis* described him as "Ioannes Suisser ... vulgo dictus Calculator", and Wallis corrected this entry to "Raimundus Suisset", the name he would have known from the 1520 edition of Swineshead's *Calculationes* in the Savile Library [Swineshead 1520 (Savile X. 6), 74; Vossius 1650 (Savile G. 21), 5].[21] Wallis also knew that Cardano had placed Suisset fourth in his list of great scientists (after Archimedes, Aristotle, and Euclid but ahead of Apollonius),[22] but despite this tribute to England from Italy, Wallis failed to include Suisset under any of the variations of his name. Richard of Wallingford has been described as "perhaps the best mathematician and astronomer of the Middle Ages" [North 1999, 33; North 1976], and his *Tractatus de sinibus demonstratis* survives in three copies in the Digby collection [MSS Digby 168, 178, 190]. Vossius omitted both Wallingford and the astronomer John Maudith, though both were recorded by Bale in his notebook and in his 1557–1559 *Catalogus* where they appear as "Ricardus Vualingforde" and "Ioannes Manduith" [Bale 1557–1559, 397, 426]. They were absent, however, from Bale's 1548 *Summarium*, which leads us to suppose that this was the edition used by Vossius. Since they escaped the attention of Vossius they were also missed by Wallis. The omission of Swineshead and Wallingford suggests that Savile's eulogy, though available to Wallis in the Savile Library, was also unknown to him.

Some of the writers mentioned by Wallis will be discussed in greater detail later in relation to Wallis's Chapter 4, but two of them, John Ashenden and Robert Holcot, will be given special mention here because Wallis himself singled them out for extra research.

> §2.25 That of *John Estwood* (or *Estwyde*, or *Eshwood*, or *Eshwid*, or *Eschuyde*,) *de Ashenden*, (or *Eshenden*, or *Ashenton*, or *Aysden*, for so many ways I find it written) I find printed at *Venice*, in the

[20] In their modern forms the names in Wallis's list at §2.24 are Clemens Langthorn, Gervase of Tilbury, Johannes Sacrobosco, Robert Grosseteste, Roger Bacon, John Pecham, Walter Odington, John Baconthorpe, Robert Holcot, John Ashenden, Richard of Kilvington, Nicholas of Lynn, John Killingworth, Richard Lavenham, Simon Bredon, John Somer, John Walter, William Batecombe, and William of Worcester or Botoner, and at §2.25 Adam Marsh, Thomas Bradwardine, and William Rede. For dates, biographies and bibliographies see [Emden 1957; Kretzmann, Kenny and Pinborg 1982, 853–892; Sharpe 1997]. References that have been found useful include: [Pedersen O. 1985 (Sacrobosco); Thomson 1940, Hunt 1955, Clanchy 1979, Southern 1986 (Grosseteste); North 1976, III, 238–270 (Odington); Xiberta 1927; North 1992b, 105–106 (Baconthorpe); Smalley 1956, Thorndike 1957, Tachau 1995 (Holcot); Snedegar 1988 (Ashenden); Kretzmann 1990 (Kilvington); North 1988, 87–133 (Lynn and Somer); North 1989a, 343–346; North 1992b, 124–127 (Killingworth); Talbot 1962, Molland forthcoming (a) (Bredon); North 1986, 126–130 (Walter); North 1989a, 337–342 (Batecombe); North 1986, 186–195 (Botoner); Clagett 1959, 220–222, 230–234; North 1992a, 79–82 (Bradwardine); North 1989a, 332–336 (Rede)].

[21] Richard Swineshead was variously known as "Suuinsete," "Suiseth," or "Suisset." He was not always distinguished from his contemporaries Roger and John Swineshead, so that his first name sometimes appears as Ioannes, Rudiger, Reyner, or Raimundus. In Swineshead 1520, his name is given as *Ricardus* in the title but *Raimundus* in the colophon.

[22] Swineshead became better known in Italy than in England: his *Calculationes* was published at Padua c. 1477, Pavia 1498, and Venice 1520. See also [Clagett 1959, 290–304; North 1992a, 89–92; Molland forthcoming (b)].

> year 1489, under the name of *Summa Astrologiae Judicialis de Accidentibus mundi, quae Anglicana vulgo nuncupatur, Joannis Eschuidi viri Anglici, peritissimi scientiae Astrologiae*; (which I mention, because his printed name differs so much from the manuscripts.) And (for the age of it) in two ancient Manuscript Copies, I find it thus subscribed, *Completa est haec compilatio tractatus secundi summae Judicialis de Accidentibus Mundi, 18 die mensis Septembris, Anno Christi 1348*, (which I take to be the Author's own words.) And then follows, *Explicit summa Judicialis de Accidentibus Mundi secundum magistrum Johannem de Estemdene, quondam socium Aulae de Merton in Oxonia*. The one of these manuscripts is in the Bodleyan Library, the other in the *Savilian*.

John Ashenden was considered one of the great medieval astronomers [Snedegar 1988]: his works survive in many manuscripts in the Savile, Digby, Selden, and Ashmole collections and his *Summa astrologiae judicialis de accidentibus mundi* ("A summary of the judgements of astrology on the happenings of the world") was indeed printed at Venice in 1489.[23] It is not surprising that the various forms of "Eastwood" and "Ashenden" caught Wallis's attention: Emden, in his *Biographical Register of the University of Oxford*, listed 5 additional variations of "Eastwood" and no fewer than 24 of "Ashenden." Even this list is incomplete as I have found several further spellings of Ashenden not listed by either Emden or Wallis. I have attempted to correlate Wallis's spellings with those to be found in the manuscripts in order to trace his sources, but without any great success. The best identifications are the unique forms "Essomdene" (or "Estomdene") in the colophon to the second book of Ashenden's *Summa judicialis* in MS Savile 25, and "Aysden" in a later hand in the same manuscript. MS Savile 25 would be the Savilian manuscript identified by Wallis:[24] it contains the second book (only) of Ashenden's *Summa judicialis* and the colophon is as quoted [MS Savile 25, ff. 1–63].[25] The manuscript Wallis knew in the Bodleian Library is harder to identify. There are two possibilities: MS Bodl. 369 (acquired in 1607) and MS Bodl. 714 (acquired from Thomas Allen in 1601). Both are complete copies of the *Summa judicialis* and end with the colophon already cited, with the names "Esshenden" and "Eschenden" respectively. Wallis could have seen either. The form "Esshenden" corresponding to Wallis's "Eshenden" appears in its most unambiguous form in MS Bodl. 369 at f. 379^{v}. There are also copies of the *Summa judicialis* in MS Digby 159 and 225 but Wallis appears not to have considered the Digby collection as "Bodleian" manuscripts (see Section 4.10 below).

> § 2.26 And I guess, that *Robertus de Holcot* (mentioned by *Vossius*), and *Robertus de Northamptona*, (of whom, in the *Savilian* Library, we have some mathematical Tracts in MS) might be the same person, (but am not sure of it,) because I find (in the County of *Northampton*) a Village called *Holcot* (about five miles distant from the Town of *Northampton*, Northward), and another called Hulcot (about as far Southward from *Northampton*), where, within a few years last past (as I am told by one who knew the person) lived one of that name (*Hulcot* of *Hulcot*) whose ancestors had lived there for a long time; (from some of whom perhaps that place might take the name, or they from it.) Now both of these places being near to the Town of *Northampton*, and within the County, it's not at all unlikely, that (in those days, when, for want of Surnames, Men were wont to be distinguished from the places of their Birth, or of their Abode) the same person might be indifferently called *Robertus de Holcot*, (*Hulkot*, or *Holkoth*,) and *Robertus de Northamptona*.

[23] The copy known to Wallis was probably that in MS Ashmole 576.

[24] During the 17th century the Savile collection was held separately from the main Bodleian collections. It was not incorporated into the Bodleian Library until the 19th century.

[25] The colophon translates as: "This compilation of the second book of the summary of teachings was completed the 18th day of September, year of Christ 1348. Here is set forth a summary of the teachings on the happenings of the world according to master John Estomdene, sometime fellow of Merton at Oxford."

Wallis was right to remain cautious about identifying Robert of Northampton with Robert Holcot. Robert of Northampton wrote an explanation, now in MS Savile 21 [ff. 42–61^{v}], of the *Theorica planetarum* of Roger of Hereford (see § 2.21). Robert Holcot, on the other hand, was famous for 200 years for his biblical commentaries, but not for mathematics. However, Bale ascribed to Holcot a work called *De effectionibus stellarum* ("The effects of the stars") [Bale 1548, 148]. This treatise has since been discovered, and is theological rather than mathematical [Thorndike 1957; Tachau 1995, 255–265], but the title alone was sufficient for Vossius (and hence Wallis) to regard Holcot as an astronomer.

Robert Holcot was certainly associated with Northampton [Smalley 1956, 7–9]: he lived at the Dominican convent in the city from 1343 until his death from plague in 1349, and probably came originally from the village of Holcot 5 miles to the northeast (the church there still contains wall paintings from the second half of the 14th century). Vossius gave the date of Robert Holcot's death erroneously as 1376, so he too may have identified him with Robert of Northampton. Place names, as Wallis realised, can be a useful guide to medieval identity but in this case may have confused the issue. The second village, Hulcot, where Wallis made his enquiries, is now absorbed into Northampton itself. Wallis had family connections in this part of Northamptonshire[26] and his research in the area shows how far he took his interests beyond the confines of Oxford and its libraries.

§ Chapter 3. Of the Numeral Figures now in use, from whence we had them

Chapter 3 marks a distinct change in Wallis's style and method. From his sweeping overview of Greek, Arab, and English mathematics he now moved into a detailed study of a single topic: the development of the modern numeral system. This was a theme that was to occupy him in one way or another for the next 10 chapters ending with the latest advances, the development of decimal fractions and logarithms. But it was here in Chapters 3 and 4 that Wallis did some of his best research, into the origin and spread of the Hindu–Arabic numerals.

§ 3.1 Amongst the Improvements in Mathematics (and particularly in Arithmetic), which we received from the *Moors* and *Arabs*, that of the *Numeral Figures*, which we now use, is very considerable: Ten in number; 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

§ 3.2 Which though they be not just the same with those of the *Arabic*, yet they are, most of them, so little different from them, that it cannot be doubted but that our Figures are derived from theirs. And those of former times (when these Figures came first into use) were yet more like to the Arabic Figures, than those we now use, which, in process of time, are by little and little sensibly varied from what at first they were: As is manifest, if we compare those we now use, with those which were then used when Printing first came in; and much more if compared with those of ancient Manuscripts before Printing.

In Chapter 2 Wallis might have been content with such general statements, but now he began to support his claims with detailed evidence.

§ 3.3 And those of *Maximus Planudes*, (whom *Vossius* placeth about the year 1370; but *Kircher* in his *Arithmologia* thinks him to have lived about 1270, and to have dedicated some of his Works to the Emperor *Michael Palaeologus*) are almost just the same with those of the *Arabs*; of whose Arithmetick, in *Greek*, we have two Manuscript Copies in the *Bodleyan Library*.

[26] Wallis's daughter Anne married John Blencowe of Marston St. Lawrence in December 1675. Blencowe is still a familiar name in the area. His second daughter Elizabeth married William Benson of Towcester, in 1682.

Maximus Planudes (c. 1255–1310) was a Greek monk who travelled as an ambassador between Constantinople and Venice and was a prolific translator from Greek to Latin. He wrote a commentary to Books I and II of Diophantus of Alexandria, a partial copy of which survives among the Savile manuscripts [MS Savile 6, ff. 91–106].[27] The "Arithmetick" of Planudes was his Ψηφοφορία κατ' Ἰνδούς ("Indian calculation") which taught the Indian figures and methods of calculation.[28] Wallis was correct in observing that there were two copies of Planudes' Ψηφοφορία in the Bodleian Library: MS Gr. Laud 51 and MS Cromw. 12, the gifts of William Laud and Oliver Cromwell respectively. There were ironies here that Wallis can hardly have failed to notice. Laud had been executed in 1645 during the war from which Cromwell emerged triumphant. The two men stood on opposing sides of the fundamental religious and political schisms which divided England in the 1640s, but both in their turn were Chancellors of Oxford University, and their names are now engraved next to each other on the great marble slab commemorating the Bodleian Library's benefactors. It is to these two men that the Library owes its two copies of the Ψηφοφορία.

Vossius' date for Maximus Planudes is rather too late, and Wallis checked it from another source, the *Arithmologia* of the Jesuit writer Athanasius Kircher [Kircher 1665, 44–47]. Kircher's book was essentially on the magic and arcane properties of numbers but he, like Wallis, was interested in how and when the numerals had reached northern Europe, and he too identified Planudes as a source but placed him in the reign of emperor Michael III Palaeologus (d. 1282) and so dated him correctly at about 1270. Kircher's opinions on the routes and dates of transmission of the numeral system will be discussed further below.

> § 3.4 But when I speak of those Figures as brought to us from the *Arabs*, I do not so much mean those very Characters which we now use, (though it be true of them also) as of the way of Computation by them; each of them, beside their own particular value, receiving a several Denomination, according as they stand in the first, second, or third place, and so forth, as far as occasion serves, each place exceeding that below it in Decuple proportion; and then, whether we retain just the same Figures, or others somewhat varied from them, (according as the fashion of letters in divers Countries, and divers Ages, do use to vary,) it is much one.

Here Wallis made an important point: that the real advance was not in the Hindu–Arabic symbols but in the system of place-value introduced with them, "the way of computation," with its unprecedented computational power and flexibility.

> § 3.5 Before these Figures were introduced, while we had no other ways of Notation for Numbers than that of the *Latin*, by a few Numeral Letters, M D C L X V I; or of the *Greeks* by the Letters of the Alphabet, α, β, γ, δ, &c. (like as before them, the *Hebrews*, *Arabs*, and other Orientals, did also design Numbers by the Letters of their Alphabet:) The exercise of Practical Arithmetic, especially in large Numbers, was but very lame, in comparison of what now it is.

It is another sign of 17th-century Oxford's new strength in oriental studies that Wallis was familiar not only with Classical but with Arab, Hebrew, and other non western sources. As early as 1657 he had already discussed alphabetic numeral systems in Hebrew, Greek, and Latin with references to Arabic, Persian, Turkish, and even Chinese [Wallis 1657,

[27] MS Savile 6 contains Planudes' commentary on Diophantus up to Book I.16. The full commentary was first published in Diophantus 1575.

[28] The Greek word ψηφος means "pebble," the equivalent of *calculus* in Latin. Planudes' text does not appear to have been translated into English. For the Greek text and a German translation see Planudes 1865. For a French translation see Planudes 1981.

Chaps. 7–8].[29] All alphabetic systems were unwieldy for carrying out large or complex calculations but were nevertheless used successfully for hundreds of years both for recording and for basic calculations. Wallis went on to give three examples of calculations in alphabetic numerals from Greece and medieval Europe:

> § 3.6 As will appear very evident, if we look into *Eutocius* (in his Commentary on *Archimedes, De dimensione Circuli*), or other of the Ancients, to see how troublesom a thing it was with them to multiply, divide, or extract the Root of a large Number.

At about the time he was writing *A treatise of algebra* Wallis was also engaged in the publication of two works of Archimedes, the *Arenarius* and the *Dimensio circuli*, both of which he compiled and corrected in Greek and Latin from earlier editions [Wallis 1676]. Wallis's version of *Dimensio circuli* included the commentary of Eutocius (c.560 A.D.) who remarked on the difficulty of Archimedes' calculations with fractions and square roots [Heath 1931, 305–309]. The difficulty is largely inherent in the calculations themselves but can only have been exacerbated by the limitations of the available notation.

> § 3.7 And so likewise in *Bede*, or others, to see what perplex Rules they are fain to give in these cases, which are now dispatched with a great deal of ease.

The Bodleian Library now owns about 80 manuscripts of works by Bede, but most are theological and I have discovered only one that contains calculations, MS Bodl. 309, already referred to in § 2.22. The volume opens with Bede's *De ratione temporum* which described the "nature, course and end of time" and included a 532-year table of Easters (28×19-year cycles). The volume continues in the same hand with a calendar of events, followed by part of the *Arithmetica* of Boethius, but the latter starts in mid sentence at Book I, Chapter 16. No author or title is named but Wallis would almost certainly have recognized the work and would no doubt have noted the multiplication square for 1 to 10 in Roman numerals.

> § 3.8 And the like in a Fragment we have in Manuscript of the *Second* Book of *Pappus*'s Collections, which is all employed in Rules for the Practice of Multiplication of great Numbers, much like those of *Bede*.

Book I and the first 13 propositions of Book II of the *Mathematical collections* of Pappus (c. 320 AD) are lost, but a copy of the second part of Book II is in the Savile Library and was edited and published by Wallis in 1688 [MS Savile 9, ff. 41–48; Wallis 1688b].[30] In it Pappus reproduced the methods of Apollonius (c. 225 B.C.) for multiplying large numbers; he stated, for example, that 500×40 was equivalent to $(5 \times 4) \times 1000$, a fact not immediately obvious in an alphabetic system.

> § 3.9 Or if, without consulting those Authors, we do but consider which way we should go about first to design, and then to extract the Square or Cubic Root of a Number to ten or twenty places (as we now design it), if we had no other way to express it, than by those Numeral Letters, M D C L X V I.

The modern description of a number as having 10 or 20 decimal places is itself, as Wallis pointed out, a positional concept. For all the benefits of the modern number system, however, few would relish the task of calculating square roots to such a degree of accuracy without

[29] For alphabetic numeral systems in Hebrew, Greek, and Arabic and others derived from them see [Ifrah 1998, 212–247].

[30] [Jones A. 1986, 46–47] suggests that Book I is extant in Arabic. Books III–VIII were first translated and published by Commandino in 1588.

mechanical aids. Not so Wallis who wrote to Thomas Smith that on 22 December 1669: "In the dark night in bed I did extract the square root of 3, 00000, 00000, 00000, 00000, 00000, 00000, 00000, 00000 which I so found to be 1,73205, 08075, 68877, 29353 &c. And did next day commit to writing" [MS Smith 54, f. 29]. (Wallis wrote this some 12 years after the event, but there is sufficient evidence elsewhere of his prodigious powers of calculation to lend the story some credence.)

> § 3.10 'Tis true, the *Arabs* had, and yet have, a way of expressing small Numbers (in like manner as the *Greeks* or *Hebrews*) by Letters of the Alphabet. And herein they follow the order of the *Hebrew* Alphabet; which I therefore think was anciently the order also of the *Arabic* Alphabet, though later Grammarians (for putting those Letters together, whose Figures are like; and differ but in Diacritical Points) have now disposed the *Arabic* Letters in another order.

Wallis was correct in identifying the Arabic alphabetic numerals with the Hebrew equivalents, and made interesting use of this mathematical information to argue (also correctly) about the history of the Arabic alphabet. Both the Hebrew and Arabic alphabets, like almost every other alphabet now in use, were derived from the Phoenician alphabet devised in the 15th century B.C. The order of the 22 Phoenician letters was fixed as early as the 14th century B.C., and although extra letters were sometimes interspersed in other languages, the original order has remained more or less unchanged in nearly all later alphabets [Ifrah 1998, 212–213]. The main exception is the Arabic alphabet which was rearranged in the seventh or eighth century A.D. to bring together letters similarly written. This may have made the teaching of reading and writing easier but it necessitated the use of mnemonics to correlate numbers with their respective letters [Ifrah 1998, 241–244] and perhaps indirectly encouraged the adoption of the Hindu–Arabic system.

> § 3.11 But beside that, (which in great Numbers would be very troublesom) they have another way much more convenient (by Ten Numeral Characters, altering their Values according to the places wherein they stand) as now we have, and which we borrowed from them.

> § 3.12 These Figures, which are wont to be called *Numeri Barbarici*, suppose (for the year) 1676, (in opposition to what are called *Numeri Romani*, MDCLXXVI:) or *Ciphrae Saracenicae*, or *Arabicae*, (because from the *Saracens* and *Arabians* they came to us:) How long they have been in use amongst them, we cannot certainly tell; but that with the *Arabians* and *Persians* they have been much longer in use than with us, I take to be very certain.

This paragraph contains two interesting descriptions, *Barbarici* and *Saracenicae*. The first was used simply to describe what was not Roman (or Greek), and was not necessarily a term of disparagement [Moyer 1999, 479]. "Saracen" was used to describe Arabs or Muslims at the time of the Crusades (1095–1270) so its use as a description of *ciphrae* is a telling indication of another route by which the Arabic numerals may have reached northern Europe, with the crusaders returning from the eastern Mediterranean. In his loose identification of "Saracen" with "Arabian" Wallis missed the important implications of the word.

> § 3.13 Nor do the *Arabians* pretend to have been the first Authors hereof, but do ascribe them to the *Indians*, from whom they borrowed them. Of which I have (in my *Opus Arithmeticum, chap.* 31.) cited an eminent Testimony out of *Al-Sephādi*, in his Commentary on a Poem of *Tograji*, where he ascribes to the *Indians*, three things whereof they glory to have been the Inventors; the Book of Golaila *Wa-damna* of a like nature with our *Æsop*'s *Fables*;) the Game of *Chess*; and the *Numeral Figures*.

Here Wallis introduced the first of the topics he was about to explore in detail, the geographical origin of the numerals. He had, as he said, already touched on this many years before in one of the first books he wrote after becoming Savilian professor, his *Mathesis universalis sive arithmeticum opus integrum*, an introduction to arithmetic. There, as part of a discourse on geometrical progression, he had given in both Arabic and Latin the story of the inventor of the game of chess, who sought as his reward the amount of rice to be had by doubling the grains on successive squares of the chessboard [Wallis 1657b, Chap. 31]. Wallis quoted the story from the commentary of al-Safadī (1297–1363) on the *Lāmiyyat al-'Adjam* of al-Tughrā'ī (1061–1121),[31] but it is common in Arab and Persian literature. The importance of the story in Wallis's present context was that, besides the game of chess, it ascribed two other wonders to the Indians: the tale of the *Panchatantra* (the source of the Persian fable *Kalīla wa-dimna*), and the numerals together with place-value.

> § 3.14 And Maximus Planudes (in his Book before cited) calls it *Λογιστικὴ 'Ινδικὴ*, and *Ψηφοφορία κατ' 'Ινδους*, The Indian way of Computation; and says expressly, *Τὰ δὲ σχήματα καί αν' τὰ 'Ινδικὰ 'εστιν*; And these Figures are Indian Figures.

See § 3.3. In Planudes' treatise both the figures and the methods of calculation are described as Indian. Planudes began by setting out the nine integers $1, 2, 3, \ldots, 9$ (in their eastern Arabic form, identical apart from the "5" with the modern Arabic numerals) and added 0, which he called *tsifra*. He went on to explain the rules for addition, subtraction, multiplication and division. In this he was following the first great text on Indian figures, that of al-Khwārizmī, which no longer survives in Arabic but has been reconstructed from early Latin translations [Folkerts 1997, 8–25].[32] Al-Khwārizmī's treatise opened with a detailed exposition of the principles of place-value followed by instructions for addition and subtraction, doubling and halving, multiplication and division, all done first for integers and then for fractions (common and sexagesimal), and it ended with the extraction of square roots. Later writers followed a similar plan but often treated integers and fractions in separate texts. Planudes, the first known Greek writer on the Indian figures and methods, dealt only with integers and only with the four basic operations of arithmetic.

> § 3.15 And a Treatise of *Algorithm* in Verse, of *Johannes de Sacro Bosco*, (or at least subjoined to that of his in Prose, and at least as ancient as it,) begins with these two Verses:
>
> *Haec Algorismus ars praesens dicitur, in qua*
> *Talibus Indorum fruimur bis quinque Figuris, &c*

Now Wallis moved from oriental to European sources. The early western writers on the new numerals, like Planudes in the east, based their work on al-Khwārizmī's seminal text, and over the course of time his name became corrupted to *algorism* or *algorithm* which became, as here, a general title for such treatises [Allard 1987; Folkerts 1997, 6–7].[33]

One of the earliest 13th-century algorisms was the one quoted here, composed in verse and known as the *Carmen de algorismo* (song of algorithm) [Halliwell 1839, 73–83; Steele

[31] Wallis obtained the translation from Edward Pococke who translated and published the *Lāmiyyat al-'Adjam* with his own detailed commentary in 1661. See also [Toomer 1996, 247–248].

[32] The treatise is thought to have been called *Kitāb fi 'l-jam 'wa'l-tafrīq* ("Treatise on gathering [addition] and dispersion [subtraction]").

[33] The three surviving 12th-century redactions of al-Khwārizmī's text are the *Liber ysagogarum alchorismi*, *Liber alchorismi*, and *Liber pulveris*.

1922, 72–80]. Wallis was hesitant in ascribing it to Sacrobosco but correct in supposing it was "at least as ancient;" it was in fact written by a French Franciscan, Alexandre de Ville Dieu (d. 1240). Little is known about Ville Dieu[34] but he wrote a treatise on ecclesiastical computation in verse in 1200 so his algorism may be supposed to date from about the same period. It became immensely popular: there are 11 copies in the Bodleian Library, 7 in the Digby collection alone, and another in the Savile manuscripts, but it was often copied without author or title so Wallis could be forgiven for failing to identify the writer. The first few lines set out the numerals and explain the principle of place-value:[35]

Haec Algorismus ars praesens dicitur, in qua
Talibus Indorum fruimur bis quinque figuris.
0.9.8.7.6.5.4.3.2.1.
Primoque significat unum: duo vera secunda
Tertia significat tria: sic procede sinistra
Donec ad extremam venias, qua cifra vocatur;
Quae nil significat; dat significare sequenti.
Quaelibet illarum si primo limite ponas,
Simpliciter se significat: si vero secundo,
Se decies; ...

This present art is called 'algorismus,' in which
We make use of twice-five Indian figures:
0.9.8.7.6.5.4.3.2.1.
The first signifies one: two the second
The third signifies three: thus proceed left
Until you come to the end, which is called 'cifra;'
Which signifies nothing; it gives significance to what is behind it.
If you put any of these in the first place,
It signifies simply itself: if in the second,
Itself tenfold...

The birthplace of Johannes Sacrobosco (c. 1200–1244 or 1256) is uncertain but Wallis took him to be English (see § 2.23).[36] He may have studied in Oxford but spent most of his life in Paris. His *Algorismus* (or *De arte numerandi*) with the opening line *Omnia que a primeva rerum origine* was composed about 1230, a little later than Ville Dieu's *Carmen de algorismo*, and it too dealt with the topics set out by al-Khwārizmī: place-value, addition, subtraction, doubling and halving, multiplication and division, all for integers. To these Sacrobosco added cube roots and an elementary treatment of arithmetic progression. It became the most popular of the medieval algorisms and remained in use as a university text across western Europe for three centuries.[37] As such it set the pattern for all subsequent texts on arithmetic: the same material in much the same order (along with fractions) was

[34] Ville Dieu was sometimes described as *Dolensis*, which suggests that he came from the region close to Mont Dol and Mont St Michel in northern France, probably from the town now known as Villedieu-les-Poêles.

[35] Translated JS.

[36] In [Wallis 1693, 6] Wallis argued on etymological grounds that Sacrobosco came from Halifax in Yorkshire. There is no historical evidence to support this suggestion but it has since become commonplace; see [Pedersen O. 1985].

[37] There is an early English translation in [Steele 1922, 33–51]. For a modern critical edition see [Pedersen F.S. 1983].

covered, for instance, in the early chapters of Oughtred's *Clavis* last published in 1702, five hundred years after Sacrobosco and almost a thousand after al-Khwārizmī.

Sacrobosco's *Algorismus* is immediately followed by the *Carmen de algorismo* of Ville Dieu in MS Savile 17 [ff. 94^{v}–104; ff. 104–108^{v}], a volume well known to Wallis. Other Bodleian Library manuscripts in which the *Algorismus* is followed by all or part of the *Carmen de algorismo* will be discussed in § 4.10.

> § 3.16 'Tis therefore I think not to be doubted, but that we had these Figures, partly by the way of *Greece* (as those of *Maximus Planudes* a *Grecian*,) and partly by the way of *Spain* (and by this especially, and before the other) from the *Moors* there, who had them from the *Saracens* or *Arabians*, and these either from the *Indians* immediately, or at least they from the *Persians*, and these from the *Indians*.

By "Greece" Wallis meant the Greek Byzantine empire centred on Constantinople. Wallis's source for the idea of double transmission, through Byzantium and through Spain, was possibly Kircher's *Arithmologia* (see Section 3.3). Kircher, like Wallis, had taken some trouble to seek out manuscript evidence, presumably in the Vatican library, and had come to the conclusion that the numerals had arrived from Byzantium through Planudes about 1270, and from Spain through the Alphonsine tables which he dated at 1252 (but which were actually written in 1272).[38] Wallis never gave any further consideration to the eastern route but, as we shall see shortly, argued for a much earlier date than Kircher's for the transmission from Spain. Kircher, for all the wealth of resources in the Vatican, lacked the kind of texts copied and used by working mathematicians in Oxford, and now available to Wallis: it was Wallis's access to Oxford's unique heritage of medieval material which enabled him to carry his argument very much further.

> § 3.17 And to this I find the Learned *Gerard Vossius* to incline (in his Book *De Scientiis Mathematicis, chap.* 8.) rather than to that of *Dasypodius*, who thinks them derived from the Letters of the *Greek* Alphabet. And *Vossius* directs to that Rule which will soon determine it, to wit, *If any of the Oriental Nations have Letters or Figures, which do resemble those of ours, those in likelihood are the Authors of them*: Which 'tis sure enough, that those of the *Arabians* do; and that so nearly, that if they had been known to *Dasypodius*, he would not himself have doubted it.

Conrad Dasypodius, writing at the end of the 16th century, put forward the idea that the modern numerals were derived from Greek alphabetic numerals and justified it with a table comparing Greek and modern numerals [Dasypodius 1593–1596; Smith and Karpinski 1911, 33 n. 2; Ifrah 1998, 358]. Vossius discounted his theory, citing the authority of Joseph Scaliger who claimed that the modern numerals did not appear in Greek texts until well after the sack of Constantinople in 1204. Vossius instructed his readers to look instead for similarities with the shapes of oriental letters and figures. Wallis produced a table to demonstrate that the medieval and modern European numerals were related to their Arabic equivalents (he gave the eastern Arabic forms though modern numerals are actually derived from the western Arabic, or *ghubar* numerals used in Spain and north Africa)

> § 3.18 These Figures *Vossius* (in the place cited) calls *Siphers*, (*Barbaras numerorum Notas quas Siphras dicimus*, &c.) and chuseth to write it with *S* rather than *C* or *Z*, as deducing it from the *Hebrew Saphar*, (*numeravit, descripsit*,) and applies it indifferently to all those ten Characters: And so it is commonly used by many others, who call them the *Arabic*, or *Saracen, Siphers* or *Ciphers*. And amongst ourselves,

[38] The tables referred to the "Alphonsine era," which began in 1252 with the coronation of Alphonso X of Léon and Castile, but were actually compiled 1263–1272 and did not reach Paris and Oxford until about 1320. See [North 1989a, 327–359].

> to *Cipher* or to *cast Account* are used promiscuously for the skill of using these Figures. And in allusion to that general signification, I suppose, it is, that writing in obscure or unusual Characters is called, writing in *Cipher*; of which *Baptista Porta* hath a Treatise, entituled, '*De Zipheris, sive furtivis literarum notis*.' But the word *Cipher*, however now it comes to be used (synecdochically) of all the ten, yet did originally belong to what we commonly call a *Cipher*, that is, o, (which denoteth *none*;) and the *Arabs* (from whom we have it) call it *Tsiphron*, from *Tsāphera*, (i.e. *Vacuum esse, inane esse*, to be void or empty) which answers to the Hebrew *Tsaphar* (with *Tsade*) *avolavit*; not from *Saphara*, which answers to the Hebrew *Saphar* (with *Samech*) *numeravit*: And so *Maximus Planudes* writes it, and applies it particularly to that note of *Nullity*. For (having recited the nine significant Figures) he adds *Τιθέασι δέ ἕτερόν τι σχῆμα ὁ καλουσι τζίφραν, κατ' Ἰνδους σημαίνον ου' δέν. They add*, saith he, (beside those nine) *a figure, which they call* Tziphra, *which, with the* Indians, *denotes none*. And again *Ἡ δέ τζίφρα γράφεται ουτως* o; i.e. *The Tsipher is thus written*, o: And therefore I think the word is as well written with *C* as with *S*; the Letter *c* (as we in *England* commonly pronounce it before *e* and *i*) having a sound like *s*, but somewhat harder, (as when we write, or some of us, *to advise*, with *s*, but *to give advice*, with *c*;) and therefore fitter to express *ts*.

The gradual change in use of the word *cipher*, from meaning *zero*, to a general *digit* (as in the French *chiffre*), and then *to reckon* (again preserved in the French *chiffrer*), and finally to *secret writing* or *code*, is indeed a fascinating piece of etymology, and one that Wallis could hardly have resisted exploring. Note, though, that although he mentioned Baptista Porta's *De ziferis* [Baptista Porta 1591], and in 1657 had mentioned several other writers on secret codes [Wallis 1657, Chap. 9], he said nothing about his own lifelong experience as a cryptanalyst.

Wallis's discussion of the use of 'c' and 's' was also typical of him; half of his *Grammatica* [Wallis 1653, 1–67] had been devoted to the subject of pronunciation, and he was to extend the above discussion even further in the 1693 translation of *A treatise of algebra* (where the English words *to prise, to appease*, but *price, peace*, etc. stand out strangely from the Latin text). Not just spelling but meaning was at stake here, for the use of 'c' rather than 's' linked the word *cipher* with *tsaphera* (to be empty) rather than *saphara* (to count or reckon).

> § 3.19 To this way of Arithmetic, by these Numeral Figures, they give the peculiar name of *Algorism*, (a word which, I believe, is not to be found any where used more anciently, nor for any other, than this way of Practical Arithmetic,) being an Arabic name, compounded by them of their *Arabic* article *Al*, with the *Greek 'Αριθμός*, (in like manner as *Ptolemy's Almagist*, is by them so called from *Al* and *μεγίστη*) The *Arabic* name of Algorithm, or *Algorism*, being of the same age with us, as is the *Arabic* way of Calculation, or Practical Arithmetic. It was anciently called also by another name, *Abacus*; which *Lucas de Burgo* (the first printed Author of this kind) supposeth to have been corruptly spoken for *Arabicus*, as coming to us from the *Arabs*.

Wallis' derivation of *algorism* has been described as "eccentric" [Molland 1994, 217], but it is also instructive, for it shows that although Wallis recognised al-Khwārizmī as the inventor of algebra, he had lost sight of him as a writer on arithmetic. He was not alone in this. An early English translator of Sacrobosco's *Algorismus* struggled to explain the word *algorism* as deriving either from *algos* (art) and *rithmus* (number) (hence the Latin *ars numerandi*), or from *gogos* (introduction) and *rithmus*, or from a mythical Indian king Algus, the supposed inventor of the art [Steele 1922, 33].[39] All these derivations are to be found in Sacrobosco's

[39] Pages 3–32 of [Steele 1922] contain an early English commentary on Ville Dieu's *Carmen de algorismo* in which the anonymous writer expounds similar ideas about the origin of the word *algorism*: "Ther was a kyng of Inde, the quich heyth Algor, and he made this craft. And after his name he called hit algorym; or els another cause is quy it is called Algorym, for the latyn word of hit Algorismus comes of Algos, grece, quid est ars, latine, craft on Englis, and rides, quid est numerus, latine, or nombur on Englys, ... quasi ars numerandi."

original text though, apart from the last, not so explicitly stated. By comparison Wallis's suggestion is creditable, and indeed half correct, in that he identified the Arabic origin of the syllable *al*. He was also correct in recognising that the term *algorithm* came into use in Europe at the same time as the Hindu–Arabic numerals and was always specifically associated with Indian methods of calculation.

Abacus was a Latin word, derived from Greek $\acute{\alpha}\beta\alpha\xi$ for a counting board, and not a corruption of *Arabicus*. In later (16th-century) European literature the *algorists* were commonly set against the *abacists* as representatives of the new methods versus the old.[40] *Abbacus* texts, however, belonged to a different tradition which had nothing to do with the old abacus methods; rather, they arose in Italy in the 13th century from the *Liber abbaci* of Leonardo of Pisa (Fibonacci). This and succeeding texts of the same kind taught written methods of computation using Hindu–Arabic numerals, unlike the earliest algorisms which taught "dust-board" methods in which numerals were erased as calculations were performed [Van Egmond 1994, 200–209]. Hence *abbacus* arithmetic was rather closer than *algorism* to modern computation.

> § Chapter 4. How ancient the use of Numeral Figures hath been in these Parts of the World.
>
> § 4.1 As to the Time when these *Numeral Figures* began first to be in use amongst us; *Vossius* tells us (in the place cited), that they have not been in use above 350 years; at least, not 400 years at the utmost. *Non nisi anni sunt CCCL, saltem infra Quadringentes, quod eas Sifras accepimus*. which Book being written about the year 1650, (as appears by the date of the Epistle prefixed;) it is as much as to say, they were not in use till the year 1300; or, at the farthest, not before 1250.

The sentence in italics is cited directly from *De Scientiis* [Vossius 1650, 34]. Wallis might also have quoted Kircher who also argued that the numerals had arrived during the period 1250–1300 (see § 3.16). Wallis suspected a much earlier date and the problem prompted him to new research, and was the theme of this, the fourth, of his opening chapters.

> § 4.2 But I take them to be somewhat more ancient than so, perhaps not in common use, but at least in Astronomical Tables: For I suppose they were first of all admitted in the Astronomical Tables, which we transcribed from the *Moors* or *Arabs*; and afterwards, by degrees, came into common use; till at length they began to be generally used in all Arithmetical Operations, as being much more convenient for that purpose than other ways of designing Numbers.

Wallis, like Kircher, recognised that the numerals made an early appearance in astronomical tables; unlike Kircher, he recognised that the Alphonsine tables were not the first to spread beyond Spain (see § 4.8).

> § 4.3 I know that in the Editions which we now have of *Boëtius*, *Bede*, and other ancient Authors, these [Arabic] figures are now frequently used: but I do not believe they were found in the ancient Manuscript Copies, from whence these printed Copies were taken; but in those, all their Numbers were expressed by the *Latin* Numeral Letters, (and in divers ancient Manuscripts I have so seen it:) And therefore I do not bring those as an argument of their Antiquity, nor do I believe they were in use (in these western Parts) when those authors were first written.

Numerals were often changed and updated in the course of copying, not only from manuscript to print, but from manuscript to manuscript (the same thing could happen with diagrams [Netz 1999]). MS Savile 20 and MS Selden Supra 25 both contain copies of Boethius'

[40] There is a well known wood-block engraving of Lady Arithmetic presiding over a smiling algorist and a gloomy abacist in Reisch 1503 and an illustration of the *Quarrel of the Abacists and the Algorists* in Recorde 1551.

Arithmetica which use Roman figures but in both copies Arabic numerals have been added alongside or in the margins. Wallis would certainly have known the first of these and probably the second also.

In the 20th century Smith and Karpinski discussed at some length the question of whether Boethius could have known the Indian numerals by way of the trade routes from the far east [Smith and Karpinski 1911, Ch. 5], and cited this paragraph from Wallis as part of their rejection of such a hypothesis.

> § 4.4 But that they are somewhat more ancient than *Vossius* mentions, I judge for these Reasons:
>
> § 4.5 First, I find in our *Savilian* Library divers ancient Manuscripts in which these figures do occur; (in some, perpetually; in others, very frequently.) Amongst which, there be two compleat Volums of Astronomical Tables, for all the Celestial Motions, and two Calendars for the Ecclesiastical Account; all of them fairly written in excellent good Vellum, with great accurateness and cost; which I judge from divers circumstances there appearing, to have been written not long after the year 1200, at least before 1250: Beside many other Astronomical Treatises, (translated divers of them out of *Arabic*) which appear to be much about the same age.

All the works Wallis described here, tables, calendars, and other treatises, are found together in a single volume now known as MS Savile 21. This was a volume Wallis knew well: he made extensive annotations on the blank flyleaf at the front of the volume and brief notes on the corresponding pages of the texts themselves, and his annotations are all concerned with dating.

> § 4.6 But when I say, *not long after* 1200, I do not know, but some of them may have been written a good while before that time, especially those two Volums of Astronomical Tables: For they are (one or both of them) the Tables of *Arzachel*, a *Moor* in *Spain*, whom *Vossius* says to have been eminent in *Spain*, about the year 1080; (but says also, that some others judge him to have been more ancient.) His Tables are accommodated to the Meridian of *Toledo*; and were written, I presume, in *Arabic*, (because, by a *Moor*, and accommodated to the *Arabian* year,) but translated into *Latin*, and so brought into *England*, by some of ours, who went on purpose into *Spain* to learn the *Arabic* Language, and to be acquainted with this kind of Learning; which was then to be learned no where but of the *Moors*, and out of *Arabic* Authors: Which Authors were not to be understood, nor the Tables translated into *Latin*, without knowledge of the *Arabic* Figures, (or as they be there called, *Indian* figures) retained (with some little alteration) in the *Latin* Translations, which we have.

The Toledan tables were not written by Arzachel, as Wallis supposed, but were compiled between 1062 and 1078 from the earlier tables of al-Khwārizmī (c. 830), al-Battānī (c. 888) and Thābit ibn Qurra (c. 870). Arzachel (al-Zarqālī of Cordoba, d. 1100) wrote the associated *Canones tabularum*, or explanations, which are found twice in MS Savile 21 (at ff. 27–41^{v} and ff. 63–103). Before the Alphonsine tables were compiled in 1272 the Toledan tables were used throughout Europe and adapted for other centres: Marseilles (c. 1140) and Oxford (1150) [North 1986, 114–117]. Those in MS Savile 21, ff. 63–103, were translated by Robert of Chester (see § 2.17), in this case described as Robertus Cestrensis.

> § 4.7 Finding therefore, that divers of our own Nation (to say nothing of others) did on this account travel into *Spain*; as *Adelardus*, about the year 1130; and *Retinensis*, about 1140; *Shelley*, about 1145; *Morley*, about 1180; it must needs be, that these Figures were in use with us, a good while before the year 1250: And, that they came into use, at the same time with this sort of *Arabic* Learning. and those who translated the *Arabic* Authors into *Latin*, (amongst whom was *Johannes Hispanicus* or *Hispalensis*, whom *Vossius* placeth about the year 1140) must needs be thought to have made use of these figures, which we find used in the oldest Manuscripts (that I have yet seen) of the *Latin* Translations of those *Arabic* Authors.

All the authors listed here have already been discussed (§ 2.15–§ 2.20) except John of Spain (or John of Seville, fl.1133–1142), the only non-English translator of Arabic texts ever mentioned by Wallis. He was one of several Jewish scholars active in Spain and a prolific translator of astronomy and astrology from Arabic to Latin. He would have been known to Wallis from his translation of the treatise on the astrolabe of Māshā'allāh (fl. 762–c. 815), to be found in MS Savile 21 [ff. 104–115], probably the reason for Wallis adding his name here.

> § 4.8 And that not only the first Copies of these Translations, but even these particular Books, are more ancient than the *Alphonsine* Tables, (first published, as *Vossius* tells us, in the year 1270; others say, in the year 1252;) because when these were once made, those of *Arzachel* grew out of date: And whoever would be at the cost and care to have Astronomical Tables so fairly written, would chuse to have those which were latest, and reputed most accurate.

Wallis argued correctly that any copy of the Toledan tables must have been made before the Alphonsine tables superseded all others. His argument is interesting in that he here saw the tables from the point of view of those who paid for them, a useful reminder that mathematics required its patrons as well as its practitioners. This appraisal of the situation from an economic as well as intellectual perspective is another of the modern aspects of Wallis's historiography.

> § 4.9 'Tis certain also, that *Johannes de Sacro Bosco*, whom *Vossius* places about the year 1232, (and who died in the year 1256) was not only acquainted with them, but hath left one or two Treatises *De Algorismo*; shewing the use of these Figures in all parts of Arithmetic, and doth appropriate to them the name of *Algorismus*. Two copies we have of it in Manuscript; one in the *Bodleian* Library, the other in the *Savilian*: which Art he divides into nine parts; *Numeration, Addition, Subtraction, Mediation, Duplation, Multiplication, Division, Progression*, and *Extractions of Roots, Square and Cubic*; Which are there performed much in the same manner as they are at this day.

The year of Sacrobosco's death comes from his tombstone in the Convent of St Mathurin in Paris but the last three words of the date in the Latin inscription, "*M christi bis C. quarto deno quater*" are ambiguous and may be read as "four tens plus four" (giving 1244) or "four fourteens" (giving 1256). Vossius chose the second interpretation, in which Wallis followed him. Modern scholars remain uncertain and have suggested other possibilities [Pedersen O. 1985, 186–192].

There is one copy of Sacrobosco's *Algorismus* in the *Savilian* Library, in MS Savile 17 [ff. 94^{v}–104]. The identity of the manuscript Wallis described as being in the *Bodleian* Library will be discussed under § 4.10.

As has already been described at § 3.15, Sacrobosco's *Algorismus* set the pattern for all later European arithmetic texts, and Wallis would have been thoroughly familiar with the ordering of the material (apart from *mediation* and *duplation* which fell out of use as separate headings); indeed, his own *Mathesis universalis* of 1657 dealt, from Chapter 10 onwards, with the same pedagogical material, with very much greater sophistication and detail but in the same order [Wallis 1657, Chaps. 10–34].

> § 4.10 And to this Treatise in Prose, there is (in both Copies) subjoyned another in Verse (as was the fashion of those times) to the same purpose: which therefore I judge to be his also, though his Name be not put to it; and if not, 'tis at least as ancient: for his in Prose cites this in Verse.

The juxtaposition of prose and verse enables us to identify the copy that Wallis said was in the *Bodleian* Library since Sacrobosco's *Algorismus* is followed by Ville Dieu's *Carmen*

de algorismo in MS Bodl. 177 and MS Digby 190.[41] In MS Bodl. 177 the two texts are interwoven with each chapter of the *Algorismus* followed by the corresponding verse of the *Carmen de algorismo*, though only up to the fifth verse where the writing breaks off and a blank page still awaits completion [MS Bodl. 177, ff. 45–45^{v}]; in MS Digby 190 the *Algorismus* [ff. 169^{v}–175] is followed by the first two verses of the *Carmen de algorismo* [f. 175] but written in prose form. The manuscript which best fits Wallis's description is therefore MS Bodl. 177. Though Wallis was mistaken in ascribing the *Carmen de algorismo* to Sacrobosco, he was correct in his relative dating of the two pieces.

> § 4.11 Now he dying (of a good age) in the year 1256, (and being well versed in these Studies) we may well think, this Treatise might be written divers years before 1250. And though, of some other Books, where we find such Figures used, it may be thought they might possibly be used in later Transcripts, though the originals had been written with the *Roman* Numbers, (as was said before of *Boëtius*, *Bede*, and others;) yet, in these, it must needs be, that the Figures are as ancient as the original, because the scope of the Book is to teach the use of them.
>
> § 4.12 And in whatever Authors we meet with the name of *Algorism*; so old, at least, we may conclude the use of these Figures to have been.

Wallis here repeated the important historiographical point he had already made in § 3.19, that the very purpose of the *Algorisms* was to teach the methods associated with the new numerals so that the title alone can always be taken as an indication of their use.

> § 4.13 In another Book of the same author, *Johannes de Sacro Bosco*, which is *De Computo Ecclesiastico*, (of which we have an ancient Manuscript Copy, wherein these Figures are alo used,) he says expressly (which shews the time wherein it was first written) *Ab incarnatione Domini elapsi sunt 1235 anni*; and therefore more ancient than either 1300 or 1250.

Sacrobosco's *De computo ecclesiastico* (beginning *Compotus est sciencia considerans tempora*) noted the increasing error in the Julian calendar. It is to be found in MS Savile 17 [ff. 141–174^{v}], a volume in which, as in MS Savile 21, Wallis made several annotations concerned with the dating of the texts. In particular he carefully transcribed onto the flyleaf the words "*Ab incarnatione domini elapsi sunt 1235*" which appear in Sacrobosco's text.

> § 4.14 I find also by a Treatise of *Robert Grosthead* (Bishop of *Lincoln*), *De Computo Ecclesiastico*, with a Calender annexed (fairly written in an ancient Manuscript in Vellum) that they were used by him also, who flourished about the same time. He was made Bishop of *Lincoln* in the year 1235, and died in the year 1253.

Before becoming bishop of Lincoln, Grosseteste was *Magister scholarum* in Oxford (1214–1231) and *Lector* to the Oxford Franciscans (1232–1235). During his Oxford period he wrote a number of scientific treatises including his *Computus*, first written c. 1210, corrected 1215–1219 and revised again in 1244. In it he noted the discrepancies between lunar and solar time and, like Sacrobosco, suggested appropriate reforms [North 1989b, 44–46; North 1992b, 131–132].

Grosseteste's *Computus* in MS Savile 21 [ff. 127–142^{v}], beginning *Compotus est sciencia numeracionis et divisionis*) is the first revised version from 1215–1219 and has been copied using Arabic numerals. It is not known when the copy was made but there is other evidence, to be discussed in § 4.18, that Grosseteste was indeed using the new numerals by 1215.

[41] The *Algorismus* also appears with the *Carmen de algorismo* in MS Add.C.93, but this was not acquired by the Bodleian Library until the 19th century.

§ 4.15 And *Roger Bacon*, whom *Vossius* placeth about the year 1255, (a person so well skilled, and so well acquainted with *Arabic* Learning, and so intimate with the persons last mentioned, as we find him to have been) cannot be thought to have been ignorant herein.

Roger Bacon (c. 1214–1292), a great admirer of Grosseteste, argued for the usefulness of mathematics in every part of intellectual activity [Bacon 1928, 117–127; Grant 1974, 90–94], but his own contribution to the subject, his *Communia mathematica*, was of little consequence. He did, however, have a good understanding of the shortcomings of the Julian calendar and suggested some practical corrections [North 1989b, 46–48; North 1992b, 132]. His learning became almost legendary. Vossius later wrote: "He was a man both learned and subtil unto a miracle, and did such wonderfull things by the help of mathematicks that by such as were envious and ignorant he was accused of diabolicall magick," an adulatory opinion which Wallis shared. Wallis was certain that a man so learned must have known the new numerals, but his paragraph here is notable for its lack of any evidence that Bacon used them during the period of interest to Wallis, before 1250. In fact most of Bacon's scientific writings date from after 1266, too late for Wallis's purposes.

§ 4.16 And *Alexander de Villa-Dei, Dolensis*, whom *Vossius* says to have lived about the year 1240, and to have written of Arithmetic, and ecclesiastical Computation, did, I presume, therein make use of these Figures. For though I do not remember that I have seen these Books, (at least not under that name;) yet these being then in use, and so convenient for that purpose, it is not likely that he would wave them, and make use of Numeral Letters, which are much more troublesom and inconvenient.

This was Wallis's first mention by name of Alexandre Ville Dieu, whom he had found (described as *Dolensis*) in the pages of *De scientiis* [Vossius 1650, 40]. Vossius noted Ville Dieu as a writer of arithmetic, but not of verse, so that Wallis never recognized him as the writer of the *Carmen de algorismo* (which he had now cited twice). Wallis thought that he had never seen Ville Dieu's ecclesiastical computation either: he was wrong here too, as will be shown in § 4.19.

§ 4.17 We have also, in Manuscript, another Treatise of *Algorism*, of *Jordanus*, (whom *Vossius* placeth about the year 1200, and Contemporary with that *Campanus*, who wrote *De Computo Ecclesiastico*;) entituled, *Algorismus Jordani, tam in Integris quam in Fractionibus, demonstratus*; in which, the use of these Figures, and the way of numbering by them, is with great accuracy described and demonstrated. Which *Algorismus* of his is very different from his *Arithmetica*, published and illustrated by *Faber Stapulensis*; yet so, as it may very well be judged, by his manner of demonstration, to be a work of the same man. And the Manuscript it self, as appears by the hand, and by the shape of the Figures, is very ancient.

Vossius said that Campanus (of Novara, d. 1296) considered Jordanus (fl. 1220) famous for his work on the astrolabe, and that Jordanus in his treatise on weights mentioned Campanus, and hence, argued Vossius, they must have been contemporaries [Vossius 1650, 178]. He was mistaken in this since Campanus wrote his major works around 1260, some forty years later than Jordanus. However, Jordanus was so renowned for his treatise on weights that many later commentaries and treatises on the subject were wrongly ascribed to him. Wallis, as usual where he had no evidence to the contrary, followed Vossius.

The identity of Jordanus remains a subject of controversy and uncertainty [Clagett 1959, 72–73; Klein 1964]. A number of mathematical treatises are ascribed to him, of which that on weights, his *Elementa Jordani super demonstrationem ponderum* is perhaps the most important [Clagett 1959, 69–159]. The algorism ascribed to him is usually known as

Demonstratio Jordani de algorismo, with an additional section on fractions, the *Demonstratio minutiis*; both are copied in MS Savile 21 [ff. 143–150] with the heading noted by Wallis. Like Sacrobosco, Jordanus covered the operations of addition, subtraction, doubling, halving, multiplication, division, and extraction of roots, but his treatment was more formal and without examples. Jordanus' work was firmly rooted in the Euclidean tradition of stating propositions and demonstrations, and he seems to have eschewed Oriental influences for although he presented the new Arabic numerals in his *Demonstratio*, he used them very little. All the earliest extant copies of another of his works, his *De numeris datis* use Roman numerals, which are fully replaced by Arabic only in much later copies [Hughes 1981, 22–38].

The *Arithmetica* of Jordanus, also mentioned here by Wallis, was also written as a series of formal definitions and propositions [Jordanus 1974], and Wallis, in an interesting example of verifying authorship from mathematical style, noted the similarity "in manner of demonstration" between this work and the *Demonstratio Jordani de algorismo*. The *Arithmetica* became a standard source of theoretical arithmetic; Jacques Lefèvre d'Etaples (Jacob Faber Stapulensis) published the propositions with his own demonstrations in 1496, but only in recent years has it been printed in full [Lefèvre 1496; Busard 1991].

Note Wallis's introduction of yet another historical method here: dating by handwriting. Though it did not enable him to establish a precise date in this case, he did recognise the useful link between period and style.

> § 4.18 And in the same Manuscript Book, wherin that of *Jordanus*, and some other small pieces are written, I find at the end of it two Celestial Schemes, relating to the year 1216; the one of them is called *Figura Anni*, representing the Position of the Heavens on *March* 22.1216; the other, *Figura Conjunctionis Saturni* & *Martis*, shewing the Position of the Heavens at the time of that Conjunction which happened the same year, *October* 4. 1216. They are both of them described by these Numeral Figures; and, in likelihood, were calculated about that time, in order to some Astrological Predictions to made thereupon. And it so happens, that this last page of that Piece, proves to be the latter leaf of that same piece of Parchment, which begins that Book of *Algorismus Demonstratus*, and therefore later written than it.

The "manuscript book" that Wallis described here is actually written in two sections of eight pages each, all from the same parchment. Note his careful observation of the construction of the manuscript as well as its written content. The book is now incorporated into MS Savile 21 [ff. 143–160^{v}]. It begins with the *Demonstratio Jordani de algorismo*, continues with copies of astronomical treatises of Thābit ibn Qurra, and ends with horoscope diagrams, the "Celestial Schemes," for use in 1216. The untrained modern reader would have difficulty in finding, let alone understanding, the sentences that date the diagrams, but Wallis transcribed them in full into the flyleaf of MS Savile 21. The importance of the diagrams, as Wallis saw, is that they date the entire section as having been copied before 1216. Modern scholars have identified the handwriting with near certainty as that of Robert Grosseteste [Thomson 1940, 22–36; Hunt 1955, 133–134; Clanchy 1979, 128; Southern 1986, 107]. This section is evidence, therefore, that Grosseteste was familiar with the new numerals by 1216 (see § 4.14).

> § 4.19 I find them also used in an ancient Treatise of Ecclesiastical Computation, in Verse, called *Massa Computi*, of which I have seen diverse Copies in Manuscript, (and I think it is also printed:) The Verses of which, I find frequently cited in later Computists. And (though I do not know the Author) that we may not doubt the age, the Work it self declares it; for, where he teacheth how to find the Solstices and Equinoctials at that age, he tells us, that in 120 years they go back one day; and that at the birth of Christ,

the Winter Solstice was on *Christmas* day; but falling backwards one day in 120 years, and ten times 120 years (that is, 1200) being then past, it was now come back from the 25th to the 15th of *December*. His words are these:

Solstitium quinis horam praecedit in annis,
Cumque diem faciant viginti quatuor horae,
Annis viginti centumque dies datur una.
Solstitium legimus Christo nascente fuisse.
Centum viginti decies jam praeteriere
Anni. Sic denis praecedit meta diebus.

This ecclesiastical computation in verse, the *De computo ecclesiastico*, was, like the *Carmen de algorismo*, the work of Ville Dieu [Steele 1909–1940, VI, 268–283]. Here Wallis said he did not know the author, whereas in § 4.16 he had named the author but said he had never seen his work. Ville Dieu's *De computo ecclesiastico* like his *Carmen de algorismo* exists in numerous copies: Wallis certainly knew it in MSS Savile 17 [ff. 175–184^{v}] and Savile 21 [ff. 161–175]. Dating mathematical texts, as here, from their internal content is still a useful historiographical method [Van Maanen 1993].

§ 4.20 But though we may hence gather the age of this Work to have been about the year 1200; yet I confess it doth not, from here alone, follow certainly, that these Figures were then in use, however we now find them in some of those Copies which we have; for it's possible, that in the first Original, the numbers here (as well as in *Bede*'s Books, *De Computo*) might be designed by Numeral Letters: And so in one Copy I find it to be. But in others, the Numbers are designed by the Numeral Figures; and (these appearing otherwise to have been in use at that time) we may as well think, they were so used in this: Yet so, as that the Numeral Letters were in use also, as even to this day they are.

In MS Savile 17 Ville Dieu's *De computo ecclesiastico* has been copied twice, first with Roman numerals then with Arabic. As a calendrical work, unlike an algorism, it could just as well be written either way, and was perhaps originally composed using Roman numerals and updated to the Arabic system later. This presents the historian with the problems Wallis had already warned about at §4.3 in relation to the work of Bede and Boethius.

§ 4.21 Beside what hath been already said, we have also a Treatise of astronomical tables of *Robertus Cestrensis*, (according to the Doctrine of *Albategnius Aracensis*) by him accommodated to the Meridian of *London*, and adjusted to the beginning of the year 1150, beginning the year at the first of *March* (that the Intercalations in *February* might cause no disturbance in numbering the days); having before (as he there tells us) compiled a like Treatise adjusted to the Meridian of *Toledo*, (according to *Abenezra*, or *Abenarza*, whom in that he follows) beginning at *Jan.* 1.1149. (as he doth his from *March*. 1.1150.) which argues, that he lived about that time, and that these Figures were then in use; For the *Latin* Numeral Letters are altogether improper for Astronomical Tables, nor do I believe that any such were ever written by those Letters: Though some indeed have been written in the *Greek* Numeral Letters (as those of *Ptolemy*), which, though less convenient than the *Indian* Figures, are yet much fitter for that purpose than the *Latin* Letters.

Robert of Chester's translation of the canons of Arzachel in MS Savile 21 has already been noted at § 4.6. *Albategnius Aracensis*, written in the manuscript as "Albatem Haracensis" was al-Battānī (c. 888 of Harrān in Mesopotamia); *Abenezra* was Rabbi Abraham ben Meir ibn Ezra (1090–1164 of Toledo), who translated from Arabic to Hebrew and did much to disseminate Arabic scientific learning. Wallis was correct in supposing that no astronomical tables were ever compiled in Roman numerals.

Robert's translation of the *Canons* was one of the sections of MS Savile 21 annotated by Wallis with particular reference to its date: 1150 appears in Arabic in the tables themselves

but is written as *m.c.l.* in the prologue which was presumably added at the time of adaptation. Robert's name appeared as Robertus Cestrensis and Wallis never made the identification with Robertus Retinensis, the translator of the Koran (see § 2.17). He did, however, go to some trouble to identify Robert of Chester as the next two paragraphs show.

> § 4.22 I am not ignorant that *Balaeus*, amongst his Writers of an *uncertain time*, mentions one *Robertus de Cestria*; and says, that *Leland* thinks he might have lived about the time of *Richard* the Second; that is, about the year 1380. But either that must be another of that name, or else *Leland* mistakes his age: For it is not likely if he lived about 1380, he would have adjusted his Tables to a time so long past, (those for *Toledo*, to the beginning of the year 1149; and those for *London*, to the end of it;) but rather (as in such cases is usual) to his own time, (as *Prophatius Judaeus* doth his, to the year 1300, when himself lived.) Nor doth he therein take notice of the Alphonsine Tables, and divers others which were more ancient than the year 1380; but only of *Albategnius* (whom *Vossius* placeth about the year 888), and *Aben-Ezra* (whom *Vossius* placeth about the year 1145:) Nor do I find him to mention any more late [sic] than that time.

Vossius drew heavily on the work of Bale and Leland but this is the only hint that Wallis himself turned to Bale: perhaps the puzzle of Robert of Chester's identity led him to check the source directly. Bale made entries for *Robert Ketenensis* in both his *Summarium* of 1548 and his *Catalogus* of 1557–1559, describing Robert's travels, his friendship with Hermann of Carinthia and his translation of the *Koran* for Peter of Cluny [Bale 1548, 85^{v}; Bale 1557–1559, Part I, 191]. As there is no mention of mathematics in either case there was no reason for Vossius to take up the accounts. Wallis missed them altogether: he would not have thought of searching the index for *Ketenensis*, a name he never used. He did, however, find an entry in the 1557–1559 *Catalogus* (in Part II, which is indexed and paginated separately from Part I) for *Robertus Chestre, vel de Cestria* whom Bale (explicitly following Leland) placed in the reign of Richard II [Bale 1557–1559, Part II, 52]. If Wallis had turned from Leland and Bale to Vossius' third English source, John Pits, he would have seen him too struggling with the problem of Robert's identity. In the *Relationum historicarum*, after a long list of authors in chronological order, Pits added an *Appendix* of 378 further writers for whom he was uncertain of the dates. Among them were *Robertus Cestrensis* immediately followed by *Robertus Cestria* who was said to have died in 1390 [Pits 1619, 900]. Pits clearly knew little of either and seems to have confused the two. Wallis, however, realised that Robert of Cestria was far too late to be a copyist of 12th-century tables.

> § 4.23 I should rather have taken it for *Robertus Cestrensis*, made Bishop of *Chester* by *William* the Conqueror, in the year 1085 (according to *Simeon Dunelmensis*), or 1087 (according to *Rudulphus de Diceto*). Or 1088 (according to *Godwin*); whom *Dunelmensis* reckons also by the name of *Robertus Cestrensis*, as present amongst others at a Council of Bishops under *Anselm*, in the year 1102. But *Godwin* calls him *Robert de Limesey*, and says, he died in the year 1116, which is too soon for our purpose. Nor do I meet with any thing concerning his skill in Mathematics. And it is not likely that he would begin his Tables from the year 1149, or 1150, a time then to come; and therefore it must be some other of that name, somewhat later, who lived about the year 1150.

Wallis's persistence in trying to identify Robert of Chester is shown by the fact that he consulted three different historians: Simeon of Durham, Ralph de Diceto and Francis Godwin. Simeon, a precentor of Durham, and Ralph de Diceto, dean of St Paul's, were 12th-century chroniclers of English history, and the *Historia de gestis regum anglorum* (ending at 1129) of Simeon is to be found with the *Abbreviationes chronicorum* (ending at 1201) of Diceto in Roger Twysden's *Historiae anglicanae scriptores decem* [Twysden 1652]. Simeon and

Diceto were the first and fifth of the 10 medieval writers published for the first time by Twysden in this weighty but apparently very popular tome.[42] It is still to be found in Duke Humfrey's Library where Wallis probably consulted it[43] and is a good example of the new accessibility of medieval material to 17th-century historians. Frances Godwin (1562–1633) was bishop of Llandaff and then Hereford and author of *A catalogue of the bishops of England* [Godwin 1601]. The entry for Robert, called Robert Limesey, is found under the bishops of Coventry and Lichfield but indicates that he was ordained at Chester in 1088. There is no hint in any of these accounts, however, that bishop Robert travelled to Spain.

> § 4.24 And I doubt not, but if we make search in our old Manuscripts about that age, we may find the use of them in the 12th and 13th Century, if not before.
>
> § 4.25 To this, I add what I have lately seen. At the Parish of *Helmden* in *Northamptonshire*, (in the house of Mr. *W William Richards*, now Minister there) on an ancient wooden Mantle-tree to the Chimney in his Parlour, (perfectly black with age and smoke, but firm and hard,) there is carved work (well enough for that age) from the one end to the other; and about the middle of it this date, (in old Carving, not yet defaced,) A^o DO^i M^o 133. But both the Letters and Figures of an antic shape, agreeing with that age.
>
> § 4.26 So that I do not doubt, but that they have been in use amongst us in *England*, at least as long ago as the year 1133; not only in Astronomical Tables, (though first introduced on that occasion). But elsewhere also: Which is near 150 years before the time that *Vossius* mentions.

The village of Helmdon lies about 30 miles north of Oxford and 3 miles from Marston St Lawrence, the home of Wallis's daughter, Anne, after her marriage to John Blencowe in 1675. William Richards was the incumbent of Helmdon from 1675 to 1705. In addition to his careful verbal description of both the physical condition and style of the lettering Wallis arranged to have a drawing made, which he reproduced as a foldout page in *A treatise of algebra* and also published in the *Philosophical transactions* [Wallis 1683; Wallis 1685, 12–13].

Wallis's claim for such an early date triggered a controversy that went on well into 19th century. In 1800 Ralph Churton, rector in the neighboring parish of Middleton Cheney, wrote to the *Gentleman's magazine* [Churton 1800, 1232]:

> Few of your Antiquarian readers need to be informed how much the inscription on the mantle-tree in the parsonage at Helmdon, in Northamptonshire, has puzzled the learned and curious in such matters ever since the celebrated Dr Wallis gave an account of it in the *Philosophical transactions* above a century ago.

Churton provided a full size tracing of the inscriptions (considerably more accurate than the drawing published by Wallis) and concluded:

> As to the decyphering ... having carefully examined the inscription four severall times [sic], and copied on thin paper with black lead all the material parts twice as often, I am satisfied, upon the whole, that Dr Wallis gave the true reading, namely, 'An^o. Do^i. M^o. 133.'

Thirty years later, however, George Baker published the first volume of his painstakingly researched *History and antiquities of the county of Northampton* and after carefully weighing the evidence came to a different conclusion [Baker 1822–1841, I, 631; Gough 1867]:

[42] Thomas Hearne, Bodleian librarian in 1712, wrote of this book that "Even puritans displayed something like patriotic ardour in purchasing copies of this work as soon as it appeared."

[43] Duke Humfrey's Library, built in 1488, is the oldest part of the Bodleian Library. Twysden is shelved, as probably it has been since it was acquired, in the Selden End completed in 1636.

> Much disputation and ingenious conjecture have been exercised in decyphering this famous date, and 1133, 1233, 1533, and 1555 have been severally suggested. Some writers have referred the initials W.R. following the date to William Renalde or Reynolde, the rector from 1523 to 1560, and the general style of the mantle-piece, its very depressed arch, and the elongated leaves in the spandrils, certainly correspond with that period, and corroborate the supposition; whilst, on the other hand, it must be admitted that the form of the M and the connecting figures strongly favour the interpretation given by Dr. Wallis. From a careful examination of the original I am inclined to attribute this singular curiosity to the rector [Reynolde], though it must be confessed his motive for introducing a fictitious date in rude or arbitrary characters, unless to puzzle future antiquaries seems inexplicable.

Later in the 19th century the vicarage was modernized and the mantelpiece, after standing in the porch exposed to weather, was taken into the church for safekeeping. There it can still be seen, but uncertainty as to its date persists. Architectural experts argue that the carved rosettes are typical of a much later period. A recent opinion states [Tracy 2000]:

> This is a very nice bressumer but it is certainly not 12th century!
>
> The carving is of provincial quality only, and the rosettes which are the only stylistically datable feature, look to be 1400–1450. It is impossible to be more precise than that.

A second expert, however, considers that the dragon "could easily be 12th century work" and admits the possibility that the piece may have originally been a 12th century lintel converted to a mantelpiece around 1500 [Baxter 2000].

None of the claims for a later date, as Baker pointed out, offers any credible alternative reading of the carved date, or takes into account the early form of the 3s. It cannot be completely ruled out that Wallis's reading was correct and that the beam was first carved, perhaps as a roof beam or lintel, in 1133, using numerals learned in the course of the early Crusades. The rosettes, more deeply carved than the numerals, could have been added later: the beam is attractive (the "provincial quality" gives it a pleasing and homely feel) and it is easy to understand why successive generations might have put it to new use rather than see it destroyed. It is also possible that the initials read by Wallis and Baker as "W.R." could be "W.K.;" it would not be the first time that Wallis mistook a medieval looped "K" for an "R" (see § 2.17), though the alternative reading sheds little more light on the date, which for the moment must remain uncertain.

In the "Additions and Emendations" added before *A treatise of algebra* went to press Wallis gave details of another inscription, from the gate of St. Augustine's College, Bristol: a transcript made by the antiquarian Thomas Smith (1638–1710) showed the date 1140 (with the 4 written "backwards" in its 12th-century form) [Wallis 1685a, 153]. Apparently Wallis tried to confirm the inscription even as his book was nearly printed, for the final sentence of the "Emendations" reads: "Having desired some to view it ... they find the Inscription, but not the Date. Which therefore seems (by some accident) to have been defaced, since Dr. Smith saw it there."[44] The subject continued to preoccupy Wallis into old age: in 1699 (aged 83) he annotated his own copy of the Latin translation of *A treatise of algebra* to the effect that he had asked Dr. John Hall (Bishop of Bristol, 1691–1710) to look for the Bristol inscription but that it was no longer to be found [Wallis 1693, Savile Gg2, 15]. In the same margin he noted a report from one Thomas Luffkin of Colchester about a window supposedly bearing the date 1090. Luffkin's letter was published in the *Philosophical transactions* that same year, and a drawing of the window procured from him was printed the following year

[44] [Wallis 1685a, 176].

[Luffkin 1699; Wallis 1700], but there is no independent verification of this very early date, and it seems most likely that it was in fact 1494 with the 4s written in the old looped style.

The Crusades from 1095 to 1270 took large numbers of Englishmen to the eastern Mediterranean where some of them must have learned the new numerals, if only for the purposes of bargaining and trading. This could have been the origin of both the Helmdon and Bristol inscriptions, and it is curious that Wallis never considered the Crusades as important in this respect (see also § 4.30, § 4.31, § 4.33).

> § 4.27 Nor need it appear strange to any, that of this number 1133, the *Thousand* is expressed by M^o, or the word *Millesimo* (of which that is an abbreviation). And only the latter part in Figures, 133; for that was (and still is) very usual. Thus in the Treatise of *Robertus Cestrensis* above mentioned, I find it thus written; *Annus namque Solaris in tercentum* 65 *dies atque unius diei quartam partem distinguitur*. And again: *Quibus executis, hos omnes dies in* 30 *multiplica*, & *multiplicationis summam per decem millia* 631 *divide*. (Where we have *tercentum* 65, for 365; and *decem millia* 631, for 10631.) and the like elsewhere. {See the Additions, pag. 153.}

In *Additions and emendations* Wallis also noted that the mixed use of words and symbols extended into early printed texts which often followed the conventions established in manuscripts [Wallis 1685a, 153; Censorinus 1503, 93, 94, 96, 111]. In 1693 he pointed to yet further examples, from the *Musica* of Boethius and the *Astronomiae historia* of Ioan Stadius [Wallis 1693, 15; Boethius 1546; Stadius 1560, 14]. These two texts are bound together in the Savile Library [Savile W.15] and the *Musica* has been liberally annotated by Wallis with modern note names and sol-fa equivalents, evidence of the range of his interests and alertness of mind even in his late seventies.

> § 4.28 Since these things were written, I find in *P. Mabillon*'s Treatise *De re Diplomatica*, (printed at *Paris*, 1681.) *Lib.II. Cap. XXV.* §*V.* mention made of a Bull of Pope *Stephen* the Ninth, (cited out of *Ughellus*'s *Italia Sacra, Tom.I. col. 465.*) thus dated: *Data anno Incarnationis MLVII Indictione XI.* With this Note of *Mabillon; Ubi pro XI ponitur* II, *vitio librarii qui pro Romanis numeris Arabicas ciphras male expressit*.

This is a clear example of Wallis's habit of adding new material to existing writing. Most of *A treatise of algebra* was composed by 1676 but the appearance of Jean Mabillon's *De re diplomatica* in 1681 caused Wallis to write additional paragraphs. The *De re diplomatica* is an enormous volume (47cm $\times$ 27 cm; 600 pages) in which Mabillon undertook to "explain and illustrate the dates, materials and writing of ancient scribes, together with inscriptions and chronological notes as they pertain to the history, origins and learning of old times." The second half of the book contains many fine full page examples of early styles of writing and, judging from the page wear on the Bodleian's copy, has been well used. Wallis, however, quoted an example from the first part, where Mabillon discussed the method of dating years from the Incarnation. Mabillon's note reads: "where II has been written for XI, by an error on the part of the scribe who has represented Arabic figures badly as Roman numerals" [Mabillon 1681, 184]. Perhaps the most remarkable thing about this passage is that Wallis spotted it at all: it is not indexed and Wallis could only have found it if he was reading the text with considerable care.

> § 4.29 The words in Ughellus are thus: *Scriptum per manus Gregorii notarii* & *camerarii Sanctae Apostolicae sedis in mense Novembris die 19 indictione 2. Datum Romae 10 Kalendas Decembris per manus Humberti dicti Episcopi Silvae Candidae* & *Bibliothecarii Sanctae Romanae* & *Apostolicae sedis, anno Deo propitio 1057. Pontificatus Domini Stephani noni primo, inidct. 2*. Where *Mabillon*

> supposeth, that in the Original (or at least in some Copy whence this was taken) it had been written (in both places) *Indict.* 11. (in these *Arabic* figures) for *Eleven*; but the Transcriber (taking them to the *Roman* Numbers for *Two* expressed it by 2. And if indeed it were so in the Original, it is an argument that these Figures were then in use (though perhaps but rarely) in the year 1057: (Or at least in the year 1058, for so perhaps it might be written the Indiction for the year of our Lord 1057, being but 10; so that here seems to have been another mistake in the copying; where, for MLVIII, he puts 1057 instead of 1058, which might easily happen, if one of the three last strokes did in the Original begin with age to disappear; unless we chuse rather to say, that they did, at *Sept.* 25. begin to reckon a new Indiction, which was sometimes done, but not constantly, as *Mabillon* in that Chapter observed.) But this Argument is only conjectural, because we are not sure what it was in the Original.

Wallis with characteristic thoroughness went back to Mabillon's source, Ferdinando Ughelli's history of the church in Italy [Ughelli 1644–1662, I, col. 465], and gave his own lengthy explanation of how 11 might have been changed to 2, and 1058 to 1057. His interest in calendrical matters and also, perhaps, his experience as *Custos archivorum* would have made him especially sensitive to the question of when years began and ended and how they were named.

> §4.30 And *Mabillon* himself takes no notice of it: For I find him there, *Lib.II. Cap. XXVIII.* §*X. thus to speak: Invenit* [*iuverit*] *hoc loco quaedem adjicere de notis numericis, quae in consignandis Diplomatum calculis adhibitae sunt ab antiquis. Hae notae duplicis sunt generis: nempe Numeri Romani* & *Arabici, quas vulgus cifras appellat. Recentior est harum cifrarum usus, quas Arabes ab Indis seculo X, Hispanos ab Arabibus saeculo XIII, accepisse cum aliis censet Athanasius Kircherus in Arithmologia sua [Part. I. Cap. IV.] Addit Papebrochius in Propylaei, num. 19. earum usum ante bella sacra usum non fuisse* [*minime notum fuisse Europaeis*]. *Ego vero nullum deprehendi ante seculum XIV. Thus Mabillon*.
>
> §4.31 But for the Reasons above-mentioned, I take the use of them in *Europe* to have been much older than so: Not perhaps in the date of Charters and Legal Records, (for in such we find, even to this day, they are scarce admitted, our Lawyers, in their Records, constantly making use of the *Latin* Numbers, MDCLXVI;) but, at first, in Astronomical Tables, and Algorithmical Operations, and then by little and little in common use. And the *Arabs* I believe had them much earlier than the tenth Century.

This new quote from Mabillon appears in *De re diplomatica* some 30 pages after the previous one [Mabillon 1681, 214]. There are some inaccuracies in Wallis's transcription: in 1693 he changed *invenit* to *convenit* but the correct word is *iuverit*: corrections are shown in upright square brackets in §4.30. The Latin passage translates as: "It might help to add here something on the numerical notations which were used in seals on documents and in calculations from antiquity. These notations were of two kinds, Roman numerals and Arabic, commonly called ciphers. The more recent are the ciphers which, according to Athanasius Kircher in his *Arithmologia*, the Arabs received from the Indians in the 10th century and the Spanish from the Arabs in the 13th century. Pappebrochius in Propylei, No. 19, adds that their use was very little known in Europe before the holy wars [Crusades]. I myself have detected nothing before the 14th century."

Wallis's dismissive "Thus Mabillon" was for him uncharacteristically curt. His scorn perhaps prevented him from taking seriously the idea that the "holy wars" had indeed played some role in bringing the new numerals to northern Europe. For the most part, however, it was true that Mabillon's comments were of little value. The dates quoted from Kircher were at least two centuries too late,[45] and Mabillon's own observations were limited to diplomatic rather than mathematical use.

[45] The Indian figures were known in the Islamic world by 760 A.D. and there is written evidence of them in Spain in 976 A.D. See [Hill 1915; Folkerts 1997, 4–6].

It was a pity that Wallis, having read the first part of *De re diplomatica* so thoroughly, did not look more carefully at the illustrations in the second half.There he would have found an example he would surely have relished. Among the full page illustrations is an example of handwritten numerals of the 14th/15th century, from the Benedictine monastery described as *Cavensis*, probably of Cava near Salerno in southern Italy.The numerals are from chapter headings and run as follows [Mabillon 1681, 373]:

1 2 3 4 5 6 7 8 9
X *X*1 *X*2 *X*3 *X*4 . .
XXX *XXX*1 302 303 304 . .
XXXX 401 402

The Arabic numerals in the third row are meant to be read as thirty-two, thirty-three... This small example serves as a useful reminder of how slow, uneven, and sometimes how painful the spread of Arabic numerals must have been.

> § 4.32 And (if I be not mistaken or misinformed) *Hermannus Contractus* (whom *Vossius* placeth about the year 1050, and Sir *Henry Savile* in a Manuscript of his, about 1040) was acquainted with them, and taught the use of them, in his time. But I think, his figures were in shape much different from those we now use, and said to be borrowed from some *Caldean* writer, and called by names of *Caldean Extraction*. But it is not the shape of the Figures, (which vary from day to day, as the shape of Letters also doth,) but the way or manner of using them, which we are now enquiring after. Of him I find mention in an ancient Manuscript in the *Bodleian* Library, That from *Hermannus* and *Prodocimus* they had learned the *Abacus*, which is another name for *Algorismus*. Nor were they then so well skilful in Oriental Languages, but that they might easily mistake a name, and write *Caldaean* for *Arabic* author.

Hermannus Contractus (d. 1054) was a monk of the Abbey of Reichenau, now on the Swiss–German border, and was said to be frail in body, hence *contractus* or shrunken, but great in mind. Posterity regarded him as a linguist (Latin, Greek, and Arabic), poet, historian, musician, philosopher, theologian, and mathematician. He wrote a history of the world from the birth of Christ to the year of his death (it was continued by one of his disciples up to 1066), and some of his musical writing has also survived [Hermannus Contractus 1884; Migne CXVIII]. Two treatises, on the making and use of astrolabes, are ascribed to him, and if it was indeed he who wrote one or both, he was probably familiar with Arabic numerals; all the Bodleian Library copies, however, use Roman numerals [Migne CXVIII, cols. 379–412].

The description of the numerals of Hermannus as “Caldean” was perhaps a reference to the unusual symbols, apparently of oriental origin, in Hermannus’s musical writings. To support his case for Hermannus’s knowledge of Hindu–Arabic numerals Wallis turned to methods rather than forms but his claim that *abacus* was another name for *algorism* as early as the 11th century is false. At that time *abacus* would have been used only in the old way to mean a counting-board, not in the later sense of *abbacus* arithmetic (see § 3.19).

Prodocimus may have been the 15th-century Italian mathematician and astronomer Prosdocimus de Beldomandis, but if so it seems strange that he was mentioned alongside Hermannus Contractus who lived four hundred years earlier. The manuscript which connects Hermannus and Prodocimus as teachers of the abacus, and which also presumably describes the numerals of Hermannus as “Caldean”, I have been unable to trace.

§ 4.33 Upon the whole matter therefore I judge, that about the middle of the eleventh Century, or between the year of our Lord 1000, and 1100, these Numerals Figures came into use amongst us in *Europe*, together with other *Arabic* Learning; first, on account of Astronomical Tables, and other Mathematical Books, and then by little and little into common practice.

This appeared to be Wallis's final thought on the matter. But before his book finally went to press he discovered, as he thought, evidence of even earlier use of the numerals, by Gerbert (later Pope Sylvester II). Wallis did not say what prompted him to study the writings of Gerbert, but he read his letters, the *Epistolae Gerberti*, with considerable care [Masson 1611].[46] He also consulted no fewer than five different accounts of Gerbert's life and work. As a result he wrote a long piece on Gerbert which he printed in *Additions and emendations* and instructed the reader to consider it inserted at the appropriate point in the main text (it was incorporated fully in the Latin translation in 1693) [Wallis 1685a, 153–157; Wallis 1693, 16–18]. For reasons of length only the first four paragraphs are reproduced here, but they give an indication of the meticulousness of Wallis's research:

§ But, upon further Search, I find the use of these Numeral Figures to have been yet Ancienter, even in these parts of the World.

§ And, in particular, I find that one *Gerbertus* or *Gerebertus*, was skilled therein; and brought the knowledge thereof, out of *Spain*, Into *France*, in the Tenth Century: As appears by divers passages in his Epistles extant, with this title *Gerberti Epistolae* published at *Paris* in the year 1611, (in Number 160.) with an account of his Life subjoined: and again in the year 1636. (in Number 161.) to which is added a second Collection, (in Number 55.)

§ He was bred a *Monk* at *Fleury* in*France*, (*Monachus Floriacensis*,) of the Order of *Benedictines*: (as appears Epist. 70.) He was, after that, an Abbat; *Coenobii Bobiensis* (who were Benedictines also,) as we sometimes find it; or (as elsewhere) *Abbatiae Sancti Columbani* in *Italy*: As appears, Epist. 2.3.4.5.12.14.18.24.83.130. But he oft complains of his ill usage there as Epist. 5.7.11.12.14.16. 19.23.34.35.40.46.84.91.92.117.118.143. and elsewhere. He stiles himself *Scholaris* or *Scholasticus*, or *quondam Scholasticus*, epist. 7.12.143.161.

§ He was afterwards (as we find in *Baronius* and others) Archbishop of *Rhemes* in the year 992; then of *Ravenna* in the year 996; and afterwards Pope of *Rome*, in the year 998, or 999; and so died in the year 1003. Whence that verse,

Scandit ab R. *Gerbertus* R. *post Papa vigens* R.

Which we find (with some little variation) in most of those that write of him.

The biographical details given by Wallis are roughly correct: Gerbert was a Benedictine monk of Aurillac, in France, and spent three years in northern Spain as a young man. On his return he became a tutor to the sons of both Otto I, Holy Roman Emperor, and Hugh Capet, king of France, and through their patronage, he became Archbishop of Rheims, then of Ravenna, and eventually the first French pope (Sylvester II) in 999. His reputation for learning became legendary, and he has been credited with being the first to introduce the Arabic numerals to northern Europe. It now seems, however, that he knew only the signs for the numerals, without the concepts of place value required for calculation. Gerbert's numerals, known as *apices*, appeared as abacus column headings [Ifrah 1998, 581] but the actual calculations were written and performed using Roman numerals.

[46] The Bodleian Library catalogue lists a copy with shelfmark 8^0 G.16 Th. Seld, possibly the one that Wallis used, but the book cannot now be found.

Wallis followed up a number of biographical accounts. Besides Baronio [Baronio 1594–1603, X, 872–927], mentioned above, Wallis also consulted William of Malmesbury [Savile 1596, 1–98 (36–37); Wallis used the second edition, 6–174 (64–66); see also Mynors 1998, 278–295], Vincent of Burgundy (or Beauvais or Bellvacensis, 1184–1264) [Beauvais 1591, Caps. 98–101], John Brompton (fl.1436) [Twysden 1652, col. 881], and Matthias Flacius (writing in 1567) [Flacius 1560–74, VI, cols. 547–548, 659]. All of these accounts (except that of Baronio) were based on that of William of Malmesbury and repeated much the same tales of Gerbert as a practitioner of black arts and a conjurer of spirits, something he was said to have learned from the Saracens in Spain.[47] Only with the beginning of modern historical scholarship at the beginning of the 17th century did the tone of such biographies begin to change. Cardinal Baronio, attempting to redeem Catholic church history, described Gerbert as the worst pope that ever lived, but dismissed the more fanciful anecdotes as tales told by lamplight by simple girls to keep themselves awake (despite the fact they were all written and spread by men). Wallis was equally keen to distance Gerbert (and himself) from the taint of superstition, and commented that William of Malmesbury "gave no great credit" to such tales, whereas in fact William and most of his successors wrote of little else. Wallis clearly wanted to establish Gerbert's credentials as a serious scholar and to this end he noted exactly which of Gerbert's 216 letters mentioned his interest in arithmetic. He ended his account of Gerbert with the following passage [Wallis 1685a, 157]:

> § Now that which makes me give the more undoubted credit to these writers (though a great while after,) as to his skill in *Algorism* or *Abacus* so early; is the concurrence of those passages which favour it, in his own *Epistles* as yet extant. For, otherwise, it is very possible (if nothing of this kind had appeared in his own writing, or of those who were his Contemporaries,) that those who should (after one or more Hundreds of year, when the names of *Abacus* and *Algorism* were come into use) write the History of *Gerbertus*, might (by a *Prolepsis* or Anticipation) make use of one or both of those Words; which, when they wrote were used for *Arithmetick*, to express his skill in Arithmetick, (though perhaps, not this kind of Arithmetick,) though the words were not known in the time whereof they wrote. But, finding the word *Abacus* (in this sense) more than once used in his own writings; there remains no scruple but that the thing was then in use, and known to him: and therefore as before we argued about the middle of the Tenth Century; and then, by him, brought into *France*, and known then to inquisitive Learned men (those especially who had to do with Astronomical Tables) though not yet into common use amongst the ordinary sort of men, and how much earlier yet it had before been known in *Spain* (amongst the *Moors* or *Saracens*) from whence he had it; doth not appear.

Unfortunately for Wallis's argument, Gerbert actually used *abacus* in its older sense, meaning a counting-board, not the new written methods. In his enthusiasm for Gerbert, Wallis was misled here into attributing to him far greater knowledge than in fact he could have possessed.

Nevertheless, despite errors and omissions, there can be no doubt that Wallis had achieved what he set out to do and had argued convincingly against Vossius' date of 1250 for the earliest arrival of the Arabic numerals in northern Europe. From his account a picture began to emerge, quite new in its time, of the slow and uneven spread of the numerals. From Wallis's research we can surmise that the numerals were partly known to a few scholars such as Gerbert and possibly Hermannus Contractus (as no doubt to travellers and traders) from the late 10th century. Inscriptions such as those at Helmdon and Bristol could have

[47] Gerbert was supposed to have fashioned a magic head which could answer all questions. Similar tales were later associated with both Grosseteste and Bacon.

been the result of greater individual contact with Islamic culture during the Crusades (1095–1270) though Wallis rather surprisingly never suggested this. Only with the flood of new translations in the 12th century did the numerals begin to appear more commonly in written texts, particularly astronomical tables. After 1200 the numerals were brought to England by Grosseteste and others, and together with the associated algorithms, were disseminated and popularized through the widely copied texts of Jordanus, Sacrobosco, and Ville Dieu. Full acceptance of the numerals, however, was a slow and uncertain process; Roman and Arabic figures were used side by side, even mixed together, for hundreds of years, and Roman numerals have never entirely died out. Modern scholarship has, of course, added a wealth of detail to this general overview. The development and spread of the Hindu–Arabic numerals was, as one might expect, a complex process: not only was there an inevitable reluctance to abandon the long established and easily understood Roman system, but there was also deep suspicion in some quarters of an eastern and non-Christian innovation. In its broad outline, however, Wallis's account has stood the test of time.

Wallis's two final paragraphs discussed the transition from manuscript to print.

> § 4.34 But the first (I think) who hath published any thing of this nature in print, is *Lucas de Burgo*, in *Italian*, in the year 1494; and after him (as *Buteo* informs us in his *Logistica*) *Stephanus a Rupe* in *French*, with whom *Stifelius*, in his *Arithmetic*, cites also *Adam Risen*, a *German*, (and all these, with their *Algorism*, treat also of *Algebra*:) For though *Hermannus Contractus, Prodocimus of Padua, Johannes de Sacro Bosco, Jordanus Nemerarius, Leonardo de Pisanus*, and others, had written thereof before; yet that was before Printing was in use: Nor do I know (though some other of their Works be yet extant,) that their Writings on this Subject have yet been printed, but are either not extant, or only in Manuscript.

Lucas de Burgo is better known as Luca Pacioli, whose *Summa* of 1494 was among the earliest arithmetics to be published.[48] *Stephanus a Rupe* of Lyons was mentioned not only by Buteo [1559] but also by Gosselin [1577], but has previously escaped identification [Van Egmond 1988, 141–142]. It seems clear, however, that he was none other than Etienne de la Roche (*Stephanus*, Stephen, Etienne; *rupes*, rock or cliff, roche), who in 1520 published much of the work of Nicholas Chuqet. *Adam Ries* published a number of arithmetic books that taught both abacus techniques and the new Indian methods and was greatly admired by *Michael Stifel*, who cited him in his *Arithmetica integra* [Ries 1523; Stifel 1543].

Wallis was correct in supposing that in the 17th century none of the work of Hermannus, Prodocimus, or Leonardo had been printed: the music of Hermannus and the mathematics of Leonardo were published only in the 19th century. The work of Jordanus was partly published in the 15th century by Jacques Lefèvre d'Etaples (see § 4.17).

Sacrobosco, on the other hand, was not only the first but the most widely published of the medieval writers. His *Algorismus* is known to have been printed at Strasbourg in 1488, and again at Vienna in 1517, Cracow in 1521, and Venice in 1523. Wallis's next paragraph (§ 4.35) enables us to identify yet another early printed version, from 1503.

> § 4.35 Besides those above-named (and before most of them) is that of *Judocus Clichtoveus*, who in the year 1503 (and again in 1522,) published a Treatise of *Jacobus Faber Stapulensis* (whose Scholar he had been), entituled, *An Epitome or short Introduction into Boetius's Arithmetic*, with his own Commentary thereon. To which Treatise of Speculative Arithmetic, he subjoyns his own Treatise of Practical Arithmetic, or *Praxis numerandi, quem Abacum vocant*. And, to both these, one much more ancient (of

[48] The first printed arithmetic was *Larte de labbacho*, Treviso 1478, see Smith 1987.

> an Author to him unknown), with this Title, *Opusculum de praxi numerorum, quod Algorismum vocant*. Of which last, I find an ancient Manuscript Copy in the *Savilian* Library, subjoyned to that Algorism of *Sacro-Bosco*, which I judge to be much of the same Antiquity with it, (about the year 1250, or sooner) and the most ancient of any yet printed; where we see, *Clichtoveus* useth both names, of *Abacus* and *Algorismus*, for this *Praxis numerorum*, by these Numeral Figures.

As Wallis described here, Josse Clichtove in 1503 and 1522 published the *Epitome* of Jacques Lefèvre d'Etaples (Jacob Faber Stapulensis) together with his own *Praxis numerandi* [Lefèvre 1503; Clichtove 1503]. He also included the piece that he described in his 1503 preface as *opusculum de praxi numerorum* ("a small work on the practice of numbers"), an algorism *non inscite (nescio quo authore) compositus* ("not unskillfully written, whose author I do not know"). The piece opens with the words *Omnia quae a primeva rerum origine processerunt* which identify it as Sacrobosco's *Algorismus*. Wallis recognized the similarity to the text in the Savile Library [MS Savile 17, ff. 94^{v}–104] but his sentence describing his find is confusing, for when he spoke here of "that Algorism of *Sacro-Bosco*" in the Savilian Library he seems to have meant Ville Dieu's *Carmen de algorismo* which he had previously ascribed to Sacrobosco (see § 3.15.) The *Opusculum* was the true *Algorismus* of Sacrobosco and precedes the *Carmen de algorismo* in MS Savile 17. Wallis failed to recognize the author of the *Opusculum*, but his observations enable us to add Paris 1503 to the list of Sacrobosco's publication dates.

Note once again Wallis's lack of distinction between abacus and algorism, whereas in Clichtove's book the words are used in quite separate contexts.

For all its shortcomings Wallis's Chapter 4 is a noteworthy piece of original research, remarkable not only for its generally correct conclusions but for Wallis's use of a variety of historiographical methods. He examined not only the written contents of the texts at his disposal but also clues given by physical appearance; he made use of his own extensive knowledge of mathematics, etymology and Classical languages, and drew on the expertise of others in Oriental studies; and he consulted a wide range of secondary sources, from medieval English chronicles to works on music, astronomy, and cryptography. His arguments were rarely overstated, and occasionally subtle, and where there were gaps in his knowledge he was not afraid to say so. Wallis's account far surpassed that of Vossius which had led to it, not only in its conclusions but in its approach: the change in style and content in less than 30 years was little short of revolutionary.

During the 17th century, historiography, like every other intellectual discipline, changed rapidly. By comparing the approaches of Vossius and Wallis this paper has explored just one facet of such change, the historiography of mathematics. After Wallis's work, mathematics could no longer be viewed in the old way as anciently revealed knowledge, sometimes lost, sometimes rediscovered, passed essentially unchanged from one civilization to the next, but as a human endeavor influenced by culture and circumstance, in which ideas spread and took root in a complex variety of ways. Wallis, through his position in Oxford, was uniquely well placed to explore the history of mathematics in a new way, and like every historian, he owed much to those before him, in his case Leland, Bale, Allen, Bodley, Savile, Twysden, Vossius, and many others, who from the mid-16th century onward collected, recorded, preserved, or published the legacy of medieval England. Above all he was indebted, as we are still, to the medieval writers themselves, both authors and copyists. The manuscripts which have survived in Oxford and elsewhere, with their exquisite penmanship, delicate

illustrations, touches of humor, and occasional unfinished pages are a moving testimony to those who, for all the harshness and unpredictability of their lives, struggled to comprehend their world and to share their insights with others.

APPENDIX I

Seventeenth-Century Bodleian Library Collections Containing Medieval Mathematical Material

Savile Collection (1619)

When Henry Savile (1549–1622), Warden of Merton College, founded the Oxford chairs of geometry and astronomy in 1619 he also donated his personal collection of mathematical books, notes, and manuscripts for the use of the Savilian professors. The original collections consisted mainly of 16th-century printed texts and about 40 handwritten volumes on mathematics and astronomy in Greek or Latin including a few important volumes of medieval texts. All the 17th-century professors added generously to the Savile Library, making it the best collection in England, perhaps anywhere, of mathematical texts up to 1700 [see Bernard 1697]. For many years it was housed in the tower between the schools of geometry and astronomy (where the Lower Reading Room reserve desk now stands); Wallis knew it thoroughly and his annotations are to be found frequently in both its books and its manuscripts. The Savile Library was incorporated into the main Bodleian Library in the 19th century.

Digby Collection (1634)

The second great collection of mathematical manuscripts came from Sir Kenelm Digby (1603–1665), then a naval commander, later a diplomat, who was encouraged to donate it by Sir William Laud, Archbishop of Canterbury and Chancellor of the University of Oxford from 1629 to 1645. Over half of Digby's collection had been bequeathed to him by his old tutor, Thomas Allen (1542–1632), a mathematician of Trinity College and later Gloucester Hall. Allen had rescued some of the mathematical texts which Merton College was forced to dispose of after the Reformation, making the Digby collection a particularly rich source of medieval mathematics. Of its 238 volumes at least 40 contain medieval mathematical texts, making it by far the richest single collection of such material in England. The Digby collection has its own catalog [Macray 1883], and the contents are not included in the Bodleian Library's *Summary catalogue of western manuscripts*.

Laud Collection (1635–1640)

Following his encouragement to Digby, William Laud (1573–1645) gave his own collection to the Bodleian Library in four donations between 1635 and 1640, almost doubling the library's existing holdings. Greek mathematical texts were acquired for Laud by John Greaves, later Savilian Professor of Astronomy (1643–1649), who travelled to Constantinople on Laud's behalf in 1637 [Coxe 1853b].

Selden Collection (1659)

The Selden collection was the legacy of jurist John Selden (1554–1654). The manuscripts are mainly of Greek and Oriental origin, but there are also a few important medieval Latin

texts. The majority of Arabic and Persian texts were acquired from the estate of John Greaves after his death in 1652.

Ashmole Collection (1683)

The Ashmole collection was acquired by the Bodleian Library later than the others, in 1683, when Wallis had already completed *A treatise of algebra*, but it too included mathematical texts and there is some reason to suppose that Wallis consulted it. It was the personal collection of Elias Ashmole (1617–1692) and reflected his special interest in alchemy and astronomy. The original Ashmolean museum was built next door to the Bodleian Library (where it is now the Museum for the History of Science) to house Ashmole's collection of "curiosities" acquired from John Tradescant in 1659; it opened in 1683 and held the manuscripts until they were transferred to the main library in 1860 [Black 1845].

APPENDIX II

English Sources Used by Vossius

For his accounts of English writers Vossius drew especially on the work of three 16th-century English historians: John Leland, John Bale, and John Pits.

John Leland. (c. 1506–1522), born in London, was educated at Christ's College, Cambridge and All Souls, Oxford and in 1530 became chaplain and library keeper to Henry VIII. In 1533 he was made "King's Antiquary," a special appointment never made before or since, and was commissioned to search out manuscripts and artefacts in the monasteries and colleges of England, many of which were about to be closed. Leland spent the best part of the next 10 years on the work and presented an account of his journey to Henry in 1545 [Leland 1549]. He planned a full account of early English writers, but it was never published in his lifetime; he became insane and died in London in 1552. The notes of his findings, however, his *Collecteana* and *Itinerary*, were circulated, copied, and used by many later historians, and the originals were eventually acquired by the Bodleian Library in 1632 [MS 5102–5106; MS 5107–5112]. They were first edited and published by Bodleian Librarian Thomas Hearne as *Itinerary of John Leland the antiquary* in 1710 and *Collecteana* in 1715.

Leland's contemporary *John Bale* (1495–1563) began his education at the Carmelite monastery in Norwich, followed by Jesus College, Cambridge. Initially a zealous Catholic, he converted and turned to writing virulent attacks on the Catholic church, earning himself the nickname of "bilious Bale," and from 1540 spent seven years in exile in Germany. After his return to England he began to keep a detailed notebook of the names, biographical details, and works of English writers, drawing freely on the earlier findings of Leland as well as his own research. Bale was perhaps particularly familiar with writers from his own East Anglian background, and his researches help to account for the sprinkling of East Anglian names in Wallis's list: John Baconthorp, Nicholas of Lynn, and Richard Lavenham, who, like Bale, were all Carmelites, as well as the Norfolk antiquary William Botoner. Bale published two major works: his *Summarium* in 1548 and his *Catalogus* in 1557–1559 [Bale 1548; Bale 1557–19]. His notebook was eventually acquired by John Selden and given to the Bodleian Library as part of the Selden collection, and its contents were published as *Index Britannia scriptorum* in 1902 [MS Selden Supra 64; Bale 1902].

Bale's work was later taken up by *John Pits* (1560–1616), who was educated at New College, Oxford, but spent most of his life in France and Bavaria. Pits's accounts of English writers in his *Relationum historicarum de rebus Anglicis* [Pits 1619] were closely based on those in Bale's *Summarium*, though Pits greatly disliked Bale and tried to redress his religious imbalance back toward Catholicism.

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