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Mathematical modelling of a mediaeval battle: the Battle of Agincourt, 1415

Richard R. Clements [a], Roger L. Hughes [b,*]

[a] *Department of Engineering Mathematics, The University of Bristol, Bristol, UK*
[b] *Department of Civil and Environmental Engineering, The University of Melbourne, Parkville, Vic. 3010, Australia*

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Abstract

Recent developments in our ability to model mathematically the motion of crowds have led to some rather unusual applications. Here a continuum theory is used to model the Battle of Agincourt, a mediaeval battle between an English army on the one side and a combined French and Burgundian army on the other. The calculation reported here predicts that an instability of the front between the opposing armies would have developed. Such an instability is consistent with the mounds of fallen reported in the chronicles of the time but is surprisingly at variance with modern descriptions, which describe the fallen as forming a straight 'wall' running the length of the battlefield. Interestingly, the study suggests that the battle was lost by the greater army, because of its excessive zeal for combat leading to sections of it pushing through the ranks of the weaker army only to be surrounded and isolated.

Keywords: Simulation; Crowd; Pedestrians; Battlefront; Agincourt

1. Introduction

There have been dramatic changes in the way in which the motion of a crowd is modelled in recent years. No longer is the motion of a crowd modelled as a continuum by the Navier–Stokes equations of fluid mechanics or as a collection of individuals by the equations of particle-dynamics. Equations of motion have been specifically developed for the purpose from general 'rules' of behaviour in psychological and sociological studies.

The present study aims at using a modern continuum theory (see [1]) that includes contact between individuals, to model the mediaeval Battle of Agincourt in 1415 at the start of the second stage of the Hundred Year War between England and France when the idea of nationalism was young. A well-documented mediaeval battle such as Agincourt is well suited to a study of the present form. Its details are neither

* Corresponding author. Tel.: +61-3-83444793; fax: +61-3-83444614.
E-mail address: rogerh@civenv.unimelb.edu.au (R.L. Hughes).

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highly obscured by history such as the battles in the Roman era, nor by smoke and complexity such as in the Napoleonic era. It is not complicated by the vastness of its scale as are the battles of the First and Second World Wars, nor the intricacies of individual behaviour in guerrilla warfare. A good general account of the Battle of Agincourt may be found in [2]. The model developed here is extremely simple but, as will be shown, it has a predictive capacity that is capable of clarifying historical accounts.

Section 2 of the present study gives an overview of the Battle of Agincourt. Section 3 presents a calculation predicting an instability of the front between the two armies. Section 4 analyses the predictions of this calculation with reference to written historical accounts, and the conclusions are given in Section 5.

2. Description of the Battle of Agincourt

The Battle of Agincourt (France) occurred on 25 October 1415 between an English army under the direct control of Henry V and a French–Burgundian army (hereafter referred to as the 'French' army) representing the interests of Charles VI. Modern detailed descriptions of the battle can be found in the comprehensive reviews of [2] and [3], for example. Only a short account is given here.

Details of the battle vary but modern accounts suggest that the English army consisted of 1000 men-at-arms on foot and in full armour and between 5000 and 6000 archers. At the time of the battle these troops were tired both from sleep deprivation and travel. They were also wet, hungry with few rations, and many were weakened by illness especially dysentery.

By contrast the French army was much larger, consisting of two lines each of 8000 men-at-arms on foot and in full armour, supported by 8000 archers and 1000 mounted men-at-arms. The French army was in high spirit and confident of victory over an enemy that had been plundering the French countryside. Most French troops desired to be in the front line from where it would be easier to take prisoners for personal profit by ransoming.

The battle occurred between two woods approximately one kilometre apart and spanned the distance between these woods as shown in Fig. 1. The two armies were initially located a little over a kilometre apart before the English army, impatient for action because of its dysentery, advanced to within 250 m of the French army and levied wave after wave of arrows of nuisance value only into the French lines in an attempt to hasten the battle.

The French responded with a mounted charge at the English archers and an attack by their first line of unmounted men-at-arms on their English equivalents. In response to the mounted charge the English line took a step backwards thereby exposing a line of stakes driven into the ground and directed so as to impale any mounted attacker. The mounted charge of the French was thrown into confusion, and successfully repelled quickly. The resulting retreat by the mounted men-at-arms interfered with the advance of the first of the two lines of unmounted French men-at-arms. The English archers also took a toll on this first line.

At this stage of the battle the casualties to both sides were limited. Certainly, the English army had shown that it could repel the mounted French men-at-arms, and the French unmounted men-at-arms advance had been disrupted, but compared with what was to come the casualties were light.

The first line of 8000 unmounted French men-at-arms failed to inflict any substantial loss to their 1000 English equivalents. To support the first French line, the second line consisting of a further 8000 unmounted men-at-arms were advanced to attack the English. This second line also failed to overpower the 1000 English men-at-arms.

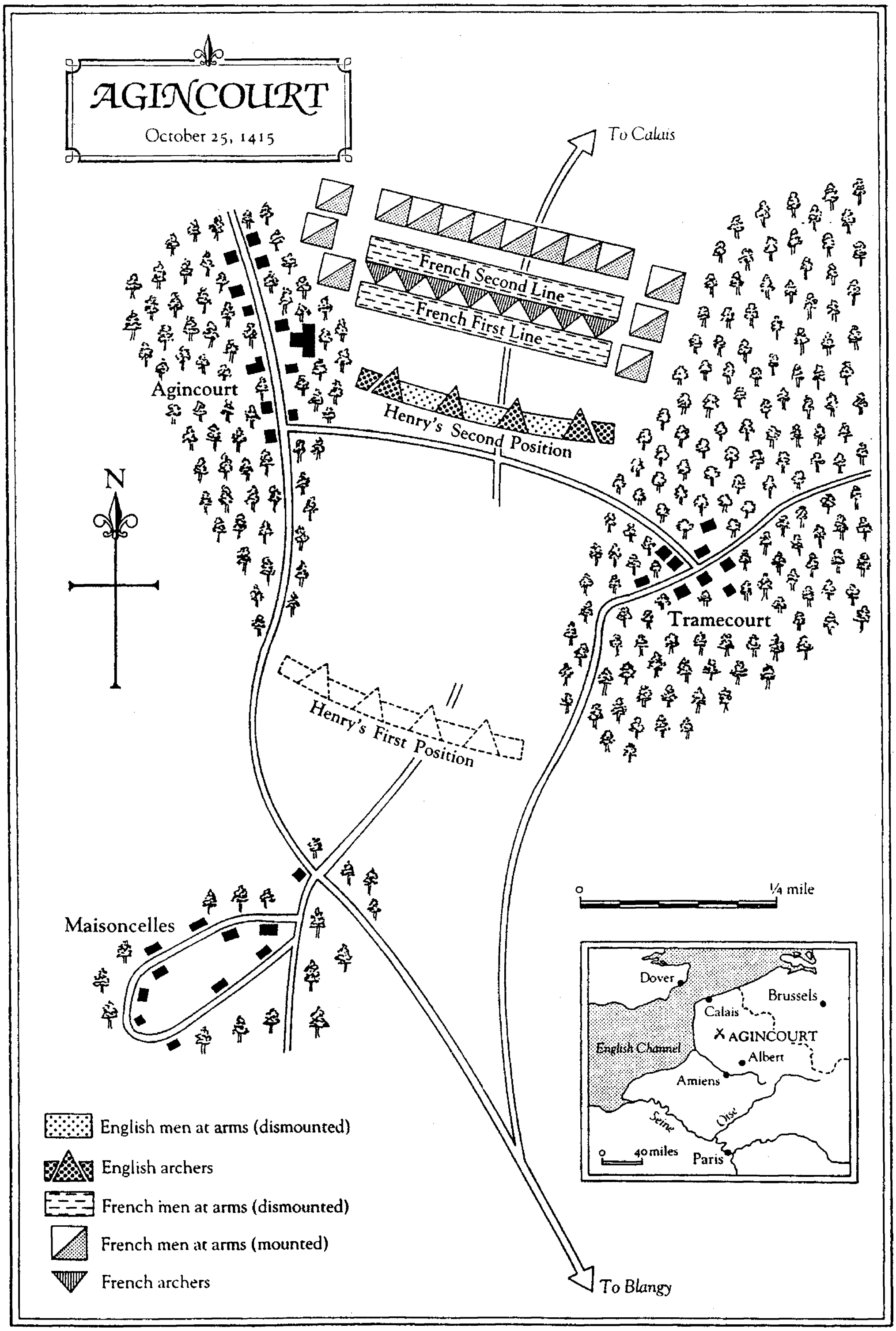

Fig. 1. Map of the battlefield as given in [3].

Several reasons have been proposed for the failure of the French men-at-arms against their English equivalents. It has been suggested that the mounted men-at-arms so disrupted their unmounted colleagues that the latter were not in a state to meet the English. However, such an explanation is not consistent with the failure of the second line. It has been suggested that crowding in the French lines resulted in insufficient room for personal manoeuvring. While there may be some validity in this argument, it cannot explain the French losses that occurred at the end of the battle. It has been suggested that the defeat occurred because the French were ill-trained. However, as the battle was fought by one-against-one conflict and in amongst the French there were many battle-hardened men-at-arms from the various conflicts that had involved France or Burgundy, there can be little doubt that many (admittedly possibly a minority) men-at-arms should have been more than a match for their tired, dysentery affected, English men-at-arms opposites. Some other explanation is needed.

At the end of the battle 6000 French men-at-arms were reportedly dead and 3000 were captive (this latter figure comes from reports that there were plans to slaughter 200 groups of 10 prisoners, of little or no value as ransom, if the battle were to continue, and 1000 additional prisoners were to be taken back to England to be held for ransom). Accepting these figures implies that approximately 7000 (determined from $2 \times 8000 - 6000 - 3000$) unmounted men-at-arms made their escape back to the French lines. Of those captured or killed, one-third were in the category of captured. This fraction is a very high proportion especially as archers, with no arrows left, were reported to have moved across the battlefield, in the latter stages of the battle, killing fallen, but not necessarily injured, French man-at-arms.

Therefore, it is natural to ask if there is likely to have been any instability of the front between the two sides of men-at-arms that could have caused the large number of prisoners to be taken. In developing a model to explore this phenomenon, it is very important to recognise that the French men-at-arms, with a desire for English prisoners to ransom, were very anxious to be at the fighting front (early in the conflict at least).

The importance of this battle is that it gave the English confidence to pursue thirty six more years of warfare in France after what had been a peaceful period within the so-called One Hundred Years War. It saw as a direct result: France ravaged by both English and French armies and its peasants in revolt; Italy pillaged by groups of discharged English armies; the English driven out of France; the English economy reduced to subsistence level to support the war effort and its peasantry in revolt; and the French king's grandson become king of English.

3. Mathematical modelling

It is emphasised here that what follows is based on the account given in Section 2 and as such is consistent with the modern interpretation of the battle but is nevertheless to some degree speculative. However, as will be seen, the modelling does give results that suggest that it is of merit.

The men-at-arms in both armies may be modelled as two human crowds. As discussed in Appendix A of [1], the equations of motion for a human crowd are

$$\frac{\partial \rho}{\partial x} = S[f(\rho)\hat{\phi}_x - u] \tag{3.1}$$

$$\frac{\partial \rho}{\partial y} = S[f(\rho)\hat{\phi}_y - v] \tag{3.2}$$

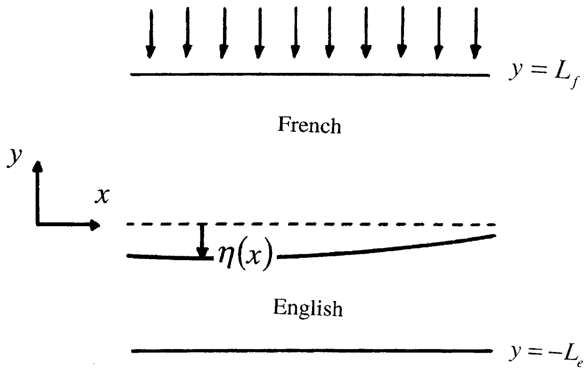

Fig. 2. Idealised representation of the battlefield as studied mathematically here.

$$\frac{\partial \rho}{\partial t}+\frac{\partial u\rho}{\partial x}+\frac{\partial v\rho}{\partial y}=0 \tag{3.3}$$

$$u\hat{\phi}_y - v\hat{\phi}_x = 0 \tag{3.4}$$

$$\hat{\phi}_x^2+\hat{\phi}_y^2=1 \tag{3.5}$$

where *S*, in general is a function of density but which is taken as constant here for simplicity as the density in the crowd of interest to us varies little proportionately. In these equations ρ, (u, v), $(\hat{\phi}_x, \hat{\phi}_y)$ and $f(\rho)$ are, respectively, the density of the crowd, the velocity field in the (x, y) direction, the direction cosines of the velocity field in the (x, y) direction and a function of the density of the crowd corresponding to the preferred speed in the absence of a density gradient. The system is free to evolve with time, t. The first two equations (Eqs. (3.1) and (3.2)) represent the (x, y) momentum equations with the left-hand-side being a measure of the contact force between pedestrians and the right-hand-side being a measure of the shear force that individuals exert on the ground. Pressure waves have been filtered from the solution by the neglecting of the time-derivative inertial terms in the full form of (3.1) and (3.2). The third equation (Eq. (3.3)), the continuity equation, represents the conservation of pedestrians. The fourth and fifth equations (Eqs. (3.4) and (3.5)), correspond to the conditions that the flow is in the direction of the direction cosines $(\hat{\phi}_x, \hat{\phi}_y)$, and that these have a vector magnitude of unity.

The present section seeks to use (3.1)–(3.5) to investigate form of the front between the English and French men-at-arms as just described assuming that the equations are valid in this situation with allowance for the direction each army faced. It should be noted that (3.1)–(3.5) were derived for very large crowds. However, the battlefront at Agincourt was typically only a dozen or so men-at-arms wide, but this number is sufficient for the model to have descriptive value as desired here. We write the thickness of this battlefront as $L = L_e + L_f$, where L_e and L_f are, respectively, the thickness of the English and French lines with $L_e \ll L_f$. Thus the English men-at-arms are between $y = -L_e$ and $y = -\eta$ and the French men-at-arms are from $y = -\eta$ to $y = L_f$, where $y = -\eta$ denotes the location of the boundary between the two armies as shown in idealised form in Fig. 2. Initially $\eta = 0$ and the boundary lies along the x-axis.

The boundary conditions applicable to (3.1)–(3.5) for each of the two armies are

$$\int_{-L_e}^{-\eta} \rho \, \mathrm{d}y = C_e \tag{3.6}$$

$$\frac{\partial \rho}{\partial y} = S_e \left[f(\rho) + \frac{\partial \eta}{\partial t} \right] \quad \text{at } y = \eta \tag{3.7}$$

for the English men-at-arms, and

$$\frac{\partial}{\partial t} \int_{-\eta}^{L_f} \rho \, \mathrm{d}y = \gamma \frac{\partial \eta}{\partial t} + \chi \quad \text{at } y = \eta \tag{3.8}$$

$$-\frac{\partial \rho}{\partial y} = S_f \left[f(\rho) - \frac{\partial \eta}{\partial t} \right] \quad \text{at } y = \eta \tag{3.9}$$

for the French men-at-arms. Here χ represents the mean flow of French men-at-arms joining the battle and the constant γ measures the local generation of a greater accumulation in those regions where the movement of the front offers the greatest opportunity for progress, that is where η is growing most rapidly. The values of S for the two armies have been given appropriate subscripts. Eqs. (3.6) and (3.8), respectively, denote the conservation of English men-at-arms, where C_e represents the density per length of the battlefront, and the accumulation of the French men-at-arms as they walk from their original position in French lines to the battle zone. Eqs. (3.7) and (3.9) state that the front moves with the speed of the men-at-arms located there. In support of (3.7) and (3.9), we note that the speed, v_0, with which men-at-arms walk forward to replace those men-at-arms who have fallen during the conflict is small in comparison with f. The speed v_0 is of the order of 10^{-3} m/s which is much less than the order of f, of order 10^0 m/s. In the absence of any fatalities v_0 would be zero, but this has no implications for the importance of higher terms in the expansion for v.

To understand the behaviour of the battlefront (3.1)–(3.9) are expanded in powers of ε where, ε represents the ratio of some typical disturbance to the density of men-at-arms to the actual density of men-at-arms at some time of interest. Thus we write

$$\rho = \rho_0 + \varepsilon \rho_1 + \varepsilon^2 \rho_2 + \cdots \tag{3.10}$$

$$u = 0 + \varepsilon u_1 + \varepsilon^2 u_2 + \cdots \tag{3.11}$$

$$v = 0 + \varepsilon v_1 + \varepsilon^2 v_2 + \cdots \tag{3.12}$$

$$\hat{\phi}_x = 0 + \varepsilon \hat{\phi}_{x1} + \varepsilon^2 \hat{\phi}_{x2} + \cdots \tag{3.13}$$

$$\hat{\phi}_y = 1 + \varepsilon \hat{\phi}_{y1} + \varepsilon^2 \hat{\phi}_{y2} + \cdots \tag{3.14}$$

where the battlefront lies along the line

$$y = \eta = 0 + \varepsilon \eta_1 + \varepsilon^2 \eta_2 + \cdots \tag{3.15}$$

and the subscripted variables on the right-hand-side of (3.10)–(3.15) denote the form of the expansion at order ε to the power of the subscript.

Substituting (3.10)–(3.15) into (3.1)–(3.9), equating powers of ε and solving for ρ_0 yields at O(ε^0)

$$\frac{d\rho_0}{dy} = +S_e f(\rho_0) \quad \text{for } y < 0 \tag{3.16}$$

$$\frac{d\rho_0}{dy} = -S_f f(\rho_0) \quad \text{for } y > 0 \tag{3.17}$$

(where

$$S_e \ll S_f \tag{3.18}$$

in response the greater resolve of the French men-at-arms).

Eqs. (3.16) and (3.17) represent two first-order equations of identical form to boundary conditions (3.7) and (3.9) at O(ε^0). Thus these first-order equations must be solved subject to the O(ε^0) expansions of (3.6) and (3.8), that is

$$\int_{-L_e}^{0} \rho_0 \, dy = C_e \tag{3.19}$$

and

$$\int_{0}^{L_f} \rho_0 \, dy = C_f \tag{3.20}$$

for the (3.16) and (3.17), respectively, where C_f denotes the linear density of French men-at-arms in the unperturbed state at a particular time of interest. Note that C_f grows linearly with time.

The form of the function $f(\rho)$ is commonly taken in the literature to be

$$f(\rho) = A - B\rho \tag{3.21}$$

where A and B are positive constants (see [4] for a summary of functions commonly used). Solving (3.16) and (3.17), with $f(\rho)$ given by (3.21), yields

$$\rho_0 = \frac{A}{B} - \left(\frac{A}{B} - \rho^-_{\text{cm}\,0}\right) \exp(-S_e B y) \quad \text{for } y < 0 \tag{3.22}$$

and

$$\rho_0 = \frac{A}{B} - \left(\frac{A}{B} - \rho^+_{\text{cm}\,0}\right) \exp(S_f B y) \quad \text{for } y > 0 \tag{3.23}$$

where ρ_{cm_0} is the mean density, $(\rho^-_{\text{cm}\,0} + \rho^+_{\text{cm}\,0})/2$, of the two opposing forces along the divide between them with superscript (−) and (+) indicating that the value of density is that located slightly inside the English and French ranks near $y = 0$. The density ρ_{cm} may be determined by (3.19) and (3.20). The values of ρ_{cm} (ρ^-_{cm} or ρ^+_{cm}), for both the English and French armies are given by

$$\rho_{\text{cm}\,0} = \frac{A}{B} - \frac{S(AL - BC)}{\exp(SBL) - 1} \tag{3.24}$$

with appropriate values for S (S_e or S_f), L (L_e or L_f) and C (C_e or C_f).

The lack of reports of one on two combat, in the references cited earlier, suggest that the density ρ_0 is nearly continuous across the front between the English and French men-at-arms. This no doubt in part is the result of limited space along the front. For our purpose we may regard the $\rho^-_{\mathrm{cm}\,0}$ and $\rho^+_{\mathrm{cm}\,0}$ as the same. As the flows involved in (3.22) and (3.23) are slow subcritical flows (see [1]), the density along the geographical divide between the solutions represents the appropriate single boundary condition in each of these first-order equations. From now on we drop the $(-)$ and $(+)$ and refer to a single value $\rho_{\mathrm{cm}\,0}$.

The density of both armies decay towards the back of the lines for each army before it also drops to zero. In the present situation the battlefront is only a dozen or so men-at-arms deep implying that $SBL \ll 1$ for both armies and so ρ_0 may be approximated by

$$\rho_0 = \rho_{\mathrm{cm}\,0} + \left(\frac{A}{B} - \rho_{\mathrm{cm}\,0}\right) S_{\mathrm{e}} B y \quad \text{for } y < 0 \tag{3.25}$$

and

$$\rho_0 = \rho_{\mathrm{cm}\,0} - \left(\frac{A}{B} - \rho_{\mathrm{cm}\,0}\right) S_{\mathrm{f}} B y \quad \text{for } y > 0 \tag{3.26}$$

The men-at-arms on both sides at $y = 0$ must withstand the stress from their fellow men-at-arms from behind. If they cannot withstand this stress they are thrust forward at their peril and the opposition men-at-arms fall back. As there is no account of the English men-at-arms uniformly retreating we discount this possibility at the $\mathrm{O}(\varepsilon^0)$ level as here. If the stress on the men-at-arms of both armies at $y = 0$ is too great the battle zone between the two armies must collapse so that neither side has space for effective use of weapons. Under such conditions the losses on both sides would be expected to be nearly the same and as they were not we discount this possibility here also.

Of more interest here is an understanding of disturbances to this density. Eqs. (3.1)–(3.5) to $\mathrm{O}(\varepsilon^1)$ yield

$$\frac{\mathrm{d}^2\rho_1}{\mathrm{d}y^2} + \frac{S_{\mathrm{e}}}{\rho_0} A \frac{\mathrm{d}\rho_1}{\mathrm{d}y} - \frac{S_{\mathrm{e}}}{\rho_0}(\lambda - S_{\mathrm{e}} B(A - B\rho_0))\rho_1 = 0 \quad \text{for } y < 0 \tag{3.27}$$

$$\frac{\mathrm{d}^2\rho_1}{\mathrm{d}y^2} - \frac{S_{\mathrm{f}}}{\rho_0} A \frac{\mathrm{d}\rho_1}{\mathrm{d}y} - \frac{S_{\mathrm{f}}}{\rho_0}(\lambda - S_{\mathrm{f}} B(A - B\rho_0))\rho_1 = 0 \quad \text{for } y > 0 \tag{3.28}$$

where the disturbance is assumes to grow as $\exp(\lambda t)$. Significantly, no derivatives in the x-direction, along the battlefront, appear as these terms are small when compared with other terms provided the distance scale of variations along the battlefront is comparable to or greater than the width of the front. Because of the significance of v at this order the equations are second-order rather than first-order as for (3.16) and (3.17).

The boundary conditions appropriate to (3.27) and (3.28) are, from (3.6)–(3.9)

$$\int_{-L_{\mathrm{e}}}^{0} \rho_1 \,\mathrm{d}y - \eta_1 \rho_0 = 0 \quad \text{at } y = 0 \tag{3.29}$$

$$\frac{\partial \rho_1}{\partial y} = S_{\mathrm{e}}(-B\rho_1 + \lambda \eta_1) \quad \text{at } y = 0 \tag{3.30}$$

for the English men-at-arms, and

$$\int_{0}^{L_{\mathrm{f}}} \rho_1 \,\mathrm{d}y + \eta_1(\rho_0 - \gamma) = 0 \quad \text{at } y = 0 \tag{3.31}$$

$$-\frac{\mathrm{d}\rho_1}{\mathrm{d}y} = S_\mathrm{f}(-B\rho_1 - \lambda\eta_1) \quad \text{at } y = 0 \tag{3.32}$$

for the French men-at-arms together with the condition that ρ_1 is continuous across the line $y = 0$ and written as $\rho_{\mathrm{cm}\,1}$ there.

It is not necessary to find the exact solution to (3.27) and (3.28), merely an accurate representation in the region $SBL < 1$. The solution in the region $-L_\mathrm{e} < y < L_\mathrm{f}$ (as for (3.25) and (3.26)) is merely

$$\rho_1 = \rho_{\mathrm{cm}\,1} - Dy \quad \text{for } y < 0 \tag{3.33}$$

and

$$\rho_1 = \rho_{\mathrm{cm}\,1} + Ey \quad \text{for } y > 0 \tag{3.34}$$

where D and E are constants. Hence (3.27)–(3.30) yield

$$L_\mathrm{e}\rho_{\mathrm{cm}\,1} + \tfrac{1}{2}L_\mathrm{e}^2 D - \rho_0\eta_1 = 0 \tag{3.35}$$

$$S_\mathrm{e}B\rho_{\mathrm{cm}\,1} - D - S_\mathrm{e}\lambda\eta_1 = 0 \tag{3.36}$$

$$L_\mathrm{f}\rho_{\mathrm{cm}\,1} + \tfrac{1}{2}L_\mathrm{f}^2 E + (\rho_0 - \gamma)\eta_1 = 0 \tag{3.37}$$

$$S_\mathrm{f}B\rho_{\mathrm{cm}\,1} - E + S_\mathrm{f}\lambda\eta_1 = 0 \tag{3.38}$$

Eqs. (3.35)–(3.38) represent four equations for the four unknown $\rho_{\mathrm{cm}\,1}$, D, E and η_1 with eigenvalue λ. For non-trivial solution this eigenvalue, the e-folding growth rate, must be

$$\lambda = \frac{-2L_\mathrm{e}(\rho_0 - \gamma) - 2L_\mathrm{f}\rho_0 - S_\mathrm{e}B(\rho_0 - \gamma)L_\mathrm{e}^2 - S_\mathrm{f}B\rho_0 L_\mathrm{f}^2}{L_\mathrm{e}L_\mathrm{f}^2 S_\mathrm{f} + L_\mathrm{e}^2 L_\mathrm{f} S_\mathrm{e} + BL_\mathrm{e}^2 L_\mathrm{f}^2 S_\mathrm{e} S_\mathrm{f}} \tag{3.39}$$

This eigenvalue is always negative for realistic values of γ, that is for values of γ that are of the same order as or smaller than ρ_0. Thus all disturbances decay and the front is stable.

However, let us suppose that the English men-at-arms cannot match the strength of the more numerous French men-at-arms. The model remains valid in all regards except that (3.30) must be replaced by

$$0 = S_\mathrm{e}(-B\rho_1 + \lambda\eta_1) \quad \text{at } y = 0 \tag{3.40}$$

English men-at-arms move to the front line to ensure that all French men-at-arms are matched in combat but the English men-at-arms are not held there by pressure from behind. Using (3.40) rather than (3.30) implies that (3.36) becomes

$$S_\mathrm{e}B\rho_{\mathrm{cm}\,1} - S_\mathrm{e}\lambda\eta_1 = 0 \tag{3.41}$$

and the eigenvalue governing the e-folding growth rate becomes

$$\lambda = \frac{-B(\rho_0 - \gamma)}{L_\mathrm{f} + BL_\mathrm{f}^2 S_\mathrm{f}} \tag{3.42}$$

which is positive for $\gamma > \rho_0$.

Thus in this case pockets of French men-at-arms are predicted to push into the English lines and with hindsight be surrounded and either taken prisoner or killed. This result is not surprising given the earlier remarks following (3.24). Indeed the above instability appears to be an unmistakable conclusion of the

assumptions of the present model provided that the assumption of the inability of the English to withstand additional pressure was correct. Such an instability might explain the victory by the weaker English army by surrounding groups of the stronger army.

However, of interest to us is that there is no preferred length scale for such incursions. The length scale is not determined by either L_e or L_f as might seem likely. Instead the length scale must be determined by external causes not within the model studied here.

The length scale along the battlefront might be thought to occur on the distance scale of the length of the battlefield or of the distance between commanding officers. However, such behaviour is not reported in the modern literature. The lack of mention of this behaviour in the modern literature suggests that the instability occurred on a length scale that is short or comparable with L_e or at the most L_f. To gain some insight into this behaviour we need to look more closely at the historic record. These difficulties concerning the modern description of the shape of the aftermath are resolved in Section 4.

4. Interpretations of predictions

Clearly, the above mathematical analysis is inconsistent with the description of the battle by modern writers, for example [2] and [3], who describe the battle as leaving a straight ‘wall’ of bodies of the slain. Thus although the model suggests that there were likely to have been many prisoners, as might be expected from an unstable battlefront, in agreement with descriptions of the battle, the model must be questioned.

However, a search of the chronicles of the time of battle gives a different picture of the battlefield [5–7]. According to these accounts the battlefield was left not with a wall of bodies where the fighting had taken place but with three large mounds of bodies. It appears that the battlefront was indeed unstable. Presumably the French men-at-arms succeeded in pushing the English men-at-arms back in certain locations as expected from the instability studied in Section 2, only to be surrounded and slaughtered. The occurrence of three mounds cannot be predicted by the model (even a large amplitude model) but appears to result from the natural tendency of troops to cluster around their leader especially in any confused situation such as a battle (note that the calculation presented here has no inherent distance scale along the battlefront).

So where did the modern picture of the battlefield originate? In his Nobel Prize (literature) winning book Canetti [8] draws attention to ‘wall’ and its derivatives of Germanic, and hence Anglo-Saxon, origin meaning ‘the dead left on the battlefield’ (see also [9]). However, the chronicles of the time were generally written in French or Latin. A search failed to find any sign of the use of a derivative of ‘wall’ and so it is concluded likely that modern accounts are not based on a mistranslation but on a more recent false description that has perpetuated without clarification by reference to the contemporary sources.

That a mathematical model is able to locate an error in modern accounts of a mediaeval battle and that that error is confirmed by looking at the original chronicles, suggests that the mathematical model is extremely applicable.

5. Conclusions

The present study has shown that mathematical modelling was able to identify an error in modern accounts of a well-documented mediaeval battle. The authors were totally unaware of this error before

the mathematical model disagreed with modern accounts. The identification of the error and its correction is of minor significance to our understanding of history but it provides an interesting demonstration of the possible importance of mathematics in this field that is normally assumed to be devoid of mathematical application.

The modern study of history is generally concerned with social issues influencing the lives of the general population not with the skirmishes of the ruling class. It remains to be seen if mathematics can usefully support the study of such social issues of historical interest. Although mathematical modelling is common in sociology, such an analysis may require further advances in this field before an adequate mathematical formulation of suitable rules of behaviour can be identified for assisting our appreciation of history.

More immediately, the present study has provided additional confirmation of the mathematical theory of crowd motion as described by [1]. It has tested the capacity of the theory in dense crowds where the motion of the crowd is blocked.

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